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COMPUTATION OF THE MAIN GEOMAGNETIC FIELD FROM SPHERICAL HARMONIC EXPANSIONS

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Computation of the Main Geomagnetic Field From Spherical Harmonic Expansions

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I. Introduction

In the past several years there has been renewed activity in the analysis of the geomagnetic field due to its importance in controlling the trajectories of cosmic rays and lower energy charged particles trapped in the magnetosphere. Since most of the interest has centered on the region from altitudes 100 to 60,000 Km, spherical harmonic representations of the geomagnetic potential have been used almost exclusively. Their usefulness in this volume of space is appropriate since the field description is smooth and is easy to drop terms from the series as they become insignificant with increasing distance from the earth. However, there are details in the computation of this field that can confuse the novice user of the numerous available computor codes since several different conventions are followed. We attempt here to point out these differences in convention and to present a coherent set of programs suitable to different uses.

II. Formulation

Making minor changes to the expressions given by Chapman and Bartels "Geomagnetism" (1940 p. 639)* we write the potential of the internal field as:

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} \left(g^{n,m} \cos m\varphi + h^{n,m} \sin m\varphi\right) P^{n,m}(\theta)$$

where a = radius of the earth

r = geocentric distance

g, h = Gauss coefficients

φ = longitude

* We will hereafter refer to this book as GM

θ = colatitude

$P^{n,m}(\theta)$ = associated Legendre functions (Gauss normalized)

from which three orthogonal components may be derived by taking the gradient

$\bar{B} = +\nabla V$ to give

$$B_\theta = \frac{\partial V}{r\,\partial\theta} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (g^{n,m}\cos m\varphi + h^{n,m}\sin m\varphi\, m)\, \frac{\partial P^{n,m}}{\partial\theta}$$

$$B_\varphi = \frac{1}{r\sin\theta}\,\frac{\partial V}{\partial\varphi} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} \frac{m}{\sin\theta}\,(-g^{n,m}\sin m\varphi + h^{n,m}\cos m\varphi)\, P^{n,m}(\theta)$$

$$B_r = \frac{\partial V}{\partial r} = -\sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (n+1)\,(g^{n,m}\cos m\varphi + h^{n,m}\sin m\varphi)P^{n,m}(\theta)$$

and $B = \sqrt{B_\theta^2 + B_\varphi^2 + B_r^2}$

We here follow the notation in GM pp. 610-611 in denoting the (Gauss-Laplace) funtions $P^{n,m}(\theta)$ which are distinct in normalization from the Neumann functions $P_{n,m}(\theta)$ and the Schmidt functions $P_n^m(\theta)$. The relations between these functions is given explicitly in GM pp. 610-611, Equations 11, 19 and 20. Here it is useful to note that

$$(2n)!/2^n n! = (2n-1)!!$$

The generating functions used to compute P and $\frac{\partial P}{\partial\theta}$ are as follows:

$$P^{o,o} = 1 \qquad \frac{\partial P^{o,o}}{\partial\theta} = 0$$

$$P^{n,n} = (\sin\theta)\, P^{n-1,\,n-1}$$

$$\frac{\partial P^{n,n}}{\partial\theta} = (\sin\theta)\,\frac{\partial P^{n-1,n-1}}{\partial\theta} + (\cos\theta)\, P^{n-1,\,n-1} \qquad (n \geq 1)$$

and for $m \neq n \geq 1$

$$P^{n,m} = (\cos\theta)\ P^{n-1,m} - K^{n,m}\ P^{n-2,m}$$

$$\frac{\partial P^{n,m}}{\partial \theta} = (\cos\theta)\ \frac{\partial P^{n-1,m}}{\partial \theta} - (\sin\theta)\ P^{n-1,m} - K^{n,m}\ \frac{\partial P^{n-2,m}}{\partial \theta} \text{ where}$$

$$K^{n,m} = \frac{(n-1)^2 - m^2}{(2n-1)(2n-3)} \qquad n > 1$$

$$K^{n,m} = 0. \qquad n = 1$$

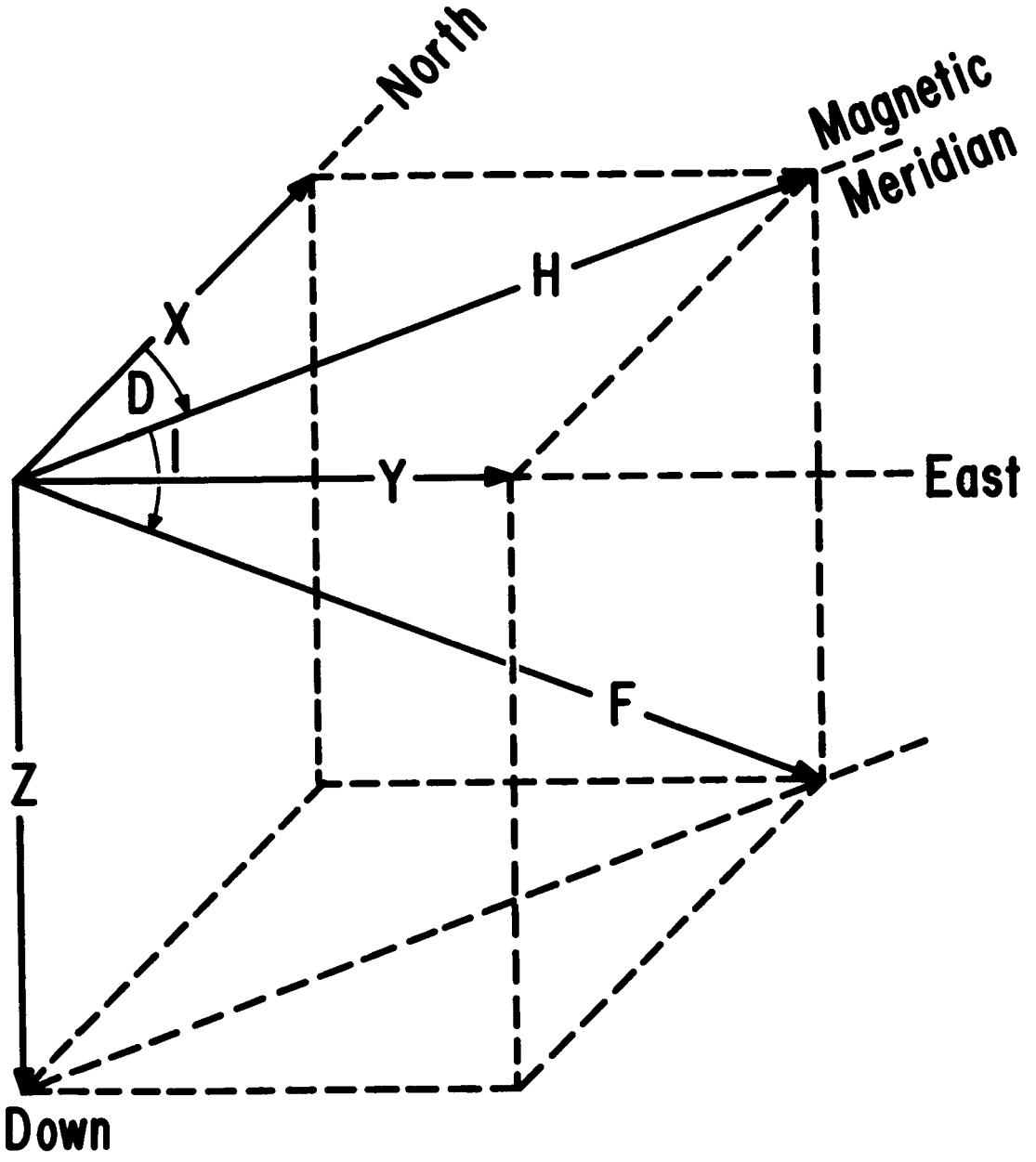
For the dipole term of degree n=1 and order m=o the previous expressions can be used to derive the field at the earths surface (r=a) as follows:

$$B_\theta = g^{1,0}\ \frac{\partial P^{1,0}}{\partial \theta} = -\ g^{1,0} \sin\theta$$

$$B_\varphi = 0$$

$$B_r = -2\ g^{1,0}\ P^{1,0} = -2g^{1,0} \cos\theta$$

Thus for $g^{1,0} > 0$, $B_\theta < 0$ for all θ and $B_r < 0$ in the northern hemisphere ($\theta < 90^\circ$). These directions are different from those conventionally used in geomagnetism as illustrated in Figure 1 (taken from GM pg. 2). In this figure the three orthogonal components of the field are X (=north), Y (= east) and Z (= downward). Except for a slight difference of definition between geodetic and geocentric coordinates to be discussed subsequently, $B_\theta \cong -X$, $B_r \cong -Z$ and $B_\varphi = Y$. We realize that in choosing to perform the evaluation of the field components utilizing $P^{n,m}(\theta)$, instead of the Schmidt quasi-normalized functions $P_n^m(\theta)$, we are creating a problem, in that by international convention the g's and h's for the expression of the geomagnetic field are to be used with the P_n^m's. Comments on this point are to be found in GM pg. 638 as follows:


North
Magnetic
Meridian
H
X
D
I
Y
East
F
Z
Down

"Workers in this field are advised to use the partly normalized functions P_n^m-----introduced by Adolf Schmidt. They have the advantage that the mean square value of P_n^m is the same as that for P_n^0, so that the coefficients in the series indicate the relative importance of the various terms. This choice would also remedy, for the future, the regrettable confusion of notation which has existed in geomagnetic literature in the past. A resolution recommending the use of Schmidt's functions was adopted by the International Association of Terrestrial Magnetism and Electricity at its Washington meeting in 1939".

The reason that the gauss normalization is adopted for computer use is that in the evaluation of B_θ, $B\varphi$ and B_r the choice of unity for the first term of the generating function saves two multiplications for each n and m. Since computer codes are being used extensively, this slight shortening of the computation time is considered worth the complication of converting g_n^m, h_n^m to $g^{n,m}$, $h^{n,m}$. We maintain the difference of convention used by Finch and Leaton (1960) whereby there is a change of sign between the two types of coefficients and a corresponding difference either in the sign of the potential V or in taking the gradient ∇V to compute the field. Thus for the earth's field the dipole term $g^{1,0} > 0$ whereas $g_1^0 < 0$.

The factors $S_{n,m}$ used for converting the "Schmidt normalized" coefficients g_n^m, h_n^m, to the "Gauss normalized" $g^{n,m}$, $h^{n,m}$ are as follows:

$$S_{0,0} = -1.$$

$$S_{n,1} = S_{n-1,0} \left[\frac{2n-1}{n}\right]$$

$$S_{n,m} = S_{n,m-1} \sqrt{\frac{(n-m+1)-J}{n+m}} \qquad \text{where } J = 2 \text{ for } m = 1, \quad J = 1 \text{ for } m > 1$$

Oblate Earth

The above formulation is rigorously correct only for a sphere. In the past geomagneticians have chosen to ignore the fact that the earth is non-spherical even to the extent of making no allowance for its oblateness. To again quote GM (pg.641):

> "While it is of considerable theoretical interest to consider the influence of the earth's oblate form, it must be admitted that this refinement has not added much to our actual knowledge of the field. It makes no serious difference to the estimates of either the exterior part, the interior part, or the non-potential part of the results. No significant physical property of the geomagnetic field can therefore be attributed to the earth's oblateness."

However, as the accuracy of evaluation of the earth's field increases it is obvious that it will eventually be essential to take the earth's true shape into account. So long as the evaluation of the harmonic coefficients is done in spherical coordinates, r, θ and φ, the resulting fields B_r, B_θ and B_φ will be in strict geocentric directions. The only constant pertaining to the earth is the radius a used in the potential function. To be consistent with past determinations we have adopted the mean value of 6371.2 Km for a. With this philosophy in mind the only problem is that of converting positions in geodetic coordinates to geocentric before evaluating the field and of transforming the resulting geocentric field vectors back to the geodetic system to compare with data measured in this system. The expressions for this conversion are given here only for the oblateness of the earth. Referring to Figure 2 we may write:

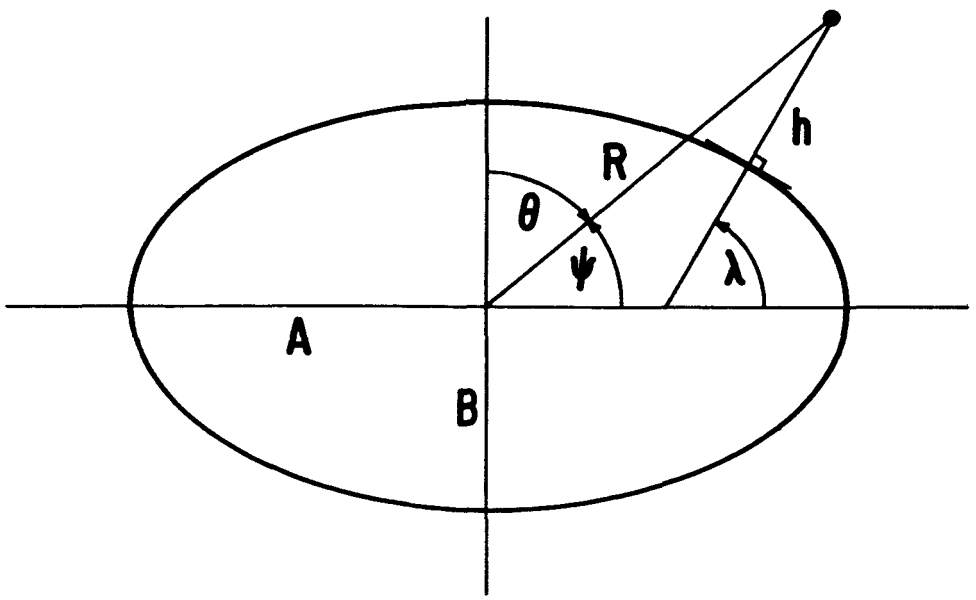


Figure 2

$$\tan\psi = \left\{ \frac{h\sqrt{A^2\cos^2\lambda + B^2\sin^2\lambda + B^2}}{h\sqrt{A^2\cos^2\lambda + B^2\sin\lambda + B^2}} \right\} \tan\lambda$$

$$\text{and } R^2 = h^2 + 2h\sqrt{A^2\cos^2\lambda + B^2\sin^2\lambda} + \frac{A^4\cos^2\lambda + B^4\sin^2\lambda}{A^2\cos^2\lambda + B^2\sin^2\lambda}$$

where:

h = height above the geoid

ψ = geocentric latitude ($90^o-\theta$)

λ = geodetic latitude

R = geocentric distance

A = equatorial radius (=6378.165 Km)

B = Polar radius (=6356.783 Km)

Using λ and h the geocentric quantities $\theta = 90 - \psi$ and R may thus be calculated for use in the previous expressions for $\bar{B}$. The conversion from B_θ, B_r to X and Y can then be done by the rotation:

$$X = -B_\theta \cos\delta \quad -B_r \sin\delta$$

$$Z = B_\theta \sin\delta \quad -B_r \cos\delta$$

where $\delta = \lambda - \psi$

For the use of coefficients of the field derived from surface data using the approximation that the earth is spherical, the proper relations are:

$$\theta = 90 - \lambda \qquad X = -B_\theta$$

$$r = 6371.2 + h \qquad Z = -B_r$$

III. General Purpose Field Program

The purpose of the first set of programs is the evaluation of the geomagnetic field components X, Y, Z, F using any available set of harmonic coefficients at any point in normal geodetic coordinates Latitude, Longitude and Altitude.

The logic of the main control program FIELDG is given in Figure 3 and the FORTRAN listing in Table I. As can be seen in the listing, the variables are:

INPUT: DLAT = Geodetic latitude λ in degrees (north positive).

DLONG = East longitude in degrees.

ALT = Height above the earth in Km.

TM = Time in years for which field is desired (e.g. July 15, 1964 = 1964.54)

NMX = Maximum degree N (=n + 1) of spherical harmonic coefficients to be used.

L = Code to read in new coefficients when > 0.

OUTPUT: X = North geodetic component (=BN)*

Y = East component (=BE)*

Z = Vertical component (=BV)*

F = Total field (=B)*

The units of the output field components will be the same as those of the coefficients.

The logical blocks of the program are thus:

(1) Set Up Constants -

On the first call of the subroutine the constants are set up for later use. The earth ellipsoid used here is that discussed by Kaula (1963), with an equatorial radius of 6378.165 Km and flattening factor of 1/298.3. (Polar radius = 6356.783 Km)

(2) Coefficient Set up -

(a.) L Test

This block branching from L < 0 reads a set of coefficients from cards in the format given in section V of this report. The L value is then internally set 0 if time terms are present or negative if not, for later use by the subroutine. This test is made only on GT (N,M) on the assumption that at least one such time derivative will be present. L should

---

*The labels BN, BE, BV, B were previously used in the Vanguard 3 Report (Cain et al, 1962) and some subsequent programs.

only be set positive by the user if a new set of coefficients is to be read in. Since L is an input parameter altered by the subroutine it should always be given as a variable in the calling sequence rather than a fixed point constant.

(b.) K Test

The branch on K=0 is the conversion of the Schmidt to Gauss normalized coefficients. For K≠0 the coefficients are assumed to be Gauss normalized and not converted.

(c.) Compute Coefficients for a new Time

The coefficients TG and TH are computed from the G's and H's after being read in (L > 0) and thereafter if L=0 and the time variable (TM) changes. Since the secular change is small, many users may only wish to change TM in increments of the order of a year.

(3) Computations

The computations from 'Comp 1' up to where the FIELD subroutine is called calculate the proper inputs for FIELD. The NMAX used in FIELD is limited to the smallest of MAXN or NMX.

(4) Field Subroutine

This subroutine is given here in two versions as listed in Tables II and IV respectively. Although the second version is discussed under the next heading, it is so constructed that it can also be called by FIELDG. The version in Table II is only a slight modification of that given in the Vanguard 3 report (Cain et al 1962). The variables are:

INPUT: ST = $\sin\theta$, CT = $\cos\theta$

SPH = $\sin\varphi$, CPH = $\cos\varphi$

R = geocentric distance (Km)

NMAX = (n + 1) maximum

OUTPUT: BT = B_θ, BP = B_φ, BR = B_r, B

The gauss normalized coefficients G and H may be fed from the TG and TH in FIELDG through COMMON. In the event that one wished to use Schmidt normalized coefficients directly instead of converting it would only be necessary to add the factor SHMIT (N,M) from the FIELDG program (after proper set-up) to the statements PNM = P (N,M) * AR and TEMP = G (N,M) * CP(M) + H(N,M) * SP (M). It is these $NMAX^2-1$ pairs of multiplications that are saved on each subroutine call by the use of gauss normalized coefficients.

One should note that although FIELD is here given as a subprogram of FIELDG, it can also appropriately be used independently provided the input variables and coefficients are in the proper form.

IV. Special Purpose Field Programs

(a) FIELD (non-indexed version):

For ultimate economy a machine-language hand coded subroutine is the most efficient. However, since the use of subscripted variables also increases execution time, particularly on some computer compilers which do not handle indexed variables very efficiently, a FIELD program with constants or constant indices could shorten the execution time. This second version of FIELD is

provided with the intent of supplying this extra efficiency. Since it is a fairly lengthy program, it is expedient to generate the FORTRAN text by the program code given in Table III. This FIELD generator is set to generate the text to NMAX = 18 but of course can be cut down to meet any specific NMAX with a resulting savings in core locations. The text actually listed in Table IV is a somewhat more compressed version of that which the program generator produces. However, the compiled version of the two programs are identical.

(b) Timing:

An approximate timing for the FIELDG subroutine for NMAX=8 and TM changing is 5.5 ms on an IBM 7094 Model I. If TM remains constant the execution time drops to about 2.5 ms. A table of the execution time for the two versions of FIELD on various machines is as follows (NMAX = 8):

	FIELD TIME (millisec)	
	indexed	non-indexed
IBM 7094 (II)	6.8	4.4
CDC 3600	8.7	6.
IBM 7094 (I)	12.4	8.3
UNIVAC 1107	16.9	12.6
IBM 1410	3700.	2500.

(c) Decrease of NMAX with h

Although the dropping of terms from the expansion with increasing R must be viewed according to the special use of the program, we include here two FORTRAN statements that may be included to cut execution time for large R:

```
NMAX = MAX1 (3., ABS (7.5/ALOG (1.001 + ALT/6371.2)*
NMAX = MIN0 (NMAX, MAXN)
```

where MAXN = maximum number of coefficients available.

Instead of performing the above computation each time one could instead of course refer to an approximate table of values of R for FIELD or ALT(Km) for FIELDG as follows:

NMAX	ALT	R/6371.2
3	71000	12.0
4	35000	6.5
5	22000	4.5
6	16000	3.5
7	12000	2.9
8	10000	2.6
9	8000	2.3
10	7000	2.1

(d) Coefficient converter

For comparison of coefficients of different normalization it may be useful to have a simple subroutine for coefficient conversion. Such a subroutine is given in Table V with formulation similar to that used in FIELDG.

*Fortran IV ALOG is $\log_e$

V. Spherical Harmonic Coefficients

Collected herein are some of the published sets of spherical harmonic coefficients in the format written by FIELDG in units of gamma (=10^{-5}Gauss). (Table VI)

VI. Sample Values

A few sample values of field generated by the FIELDG program using the set of parameters marked "April 64 coefficients" are as follows:

TM = 1960.0 (=Epoch), NMX = 8

DLAT	DLONG	ALT	X	Y	Z	F
-60	-180	0	10362	8221	-64160	65509
-60	-180	100	9822	7826	-61048	62327
-60	0	0	16356	-6532	-31525	36111
0	0	0	28047	-5969	-11352	30840
0	0	100	26710	-5695	-10446	29240
30	-60	0	22650	-6341	43057	49063
60	120	0	14060	-2976	59558	61268

VII. Acknowledgements

We appreciate the help of Dr. A. Hassitt of the University of California at San Diego in timing the FIELD programs on the CDC 3600 and of Mr. D. Parker of GSFC in providing the timing figures for the UNIVAC. Mrs. P. Conner was also very helpful in preparing the program listings.

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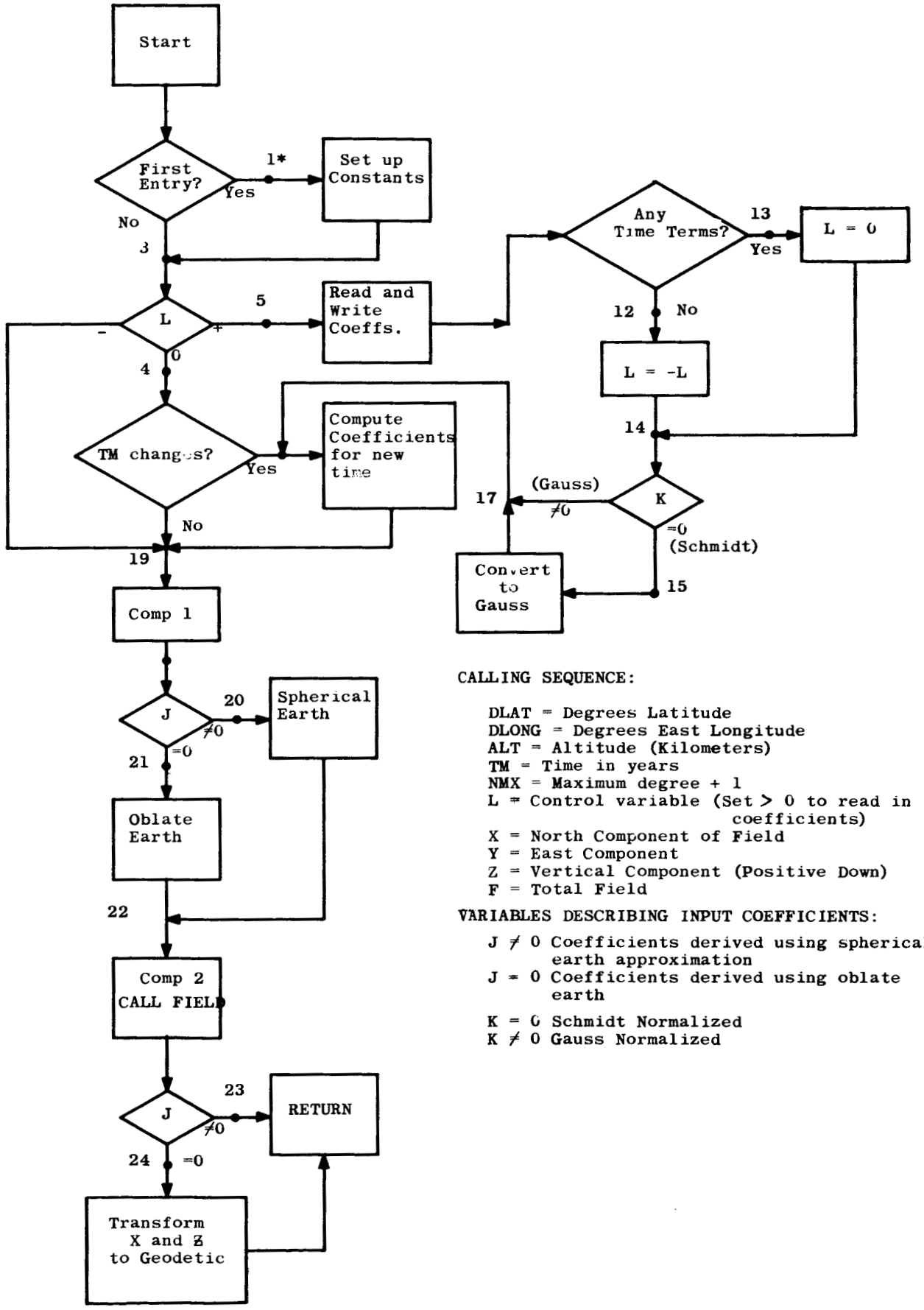
SUBROUTINE FIELDG



CALLING SEQUENCE:

DLAT = Degrees Latitude
DLONG = Degrees East Longitude
ALT = Altitude (Kilometers)
TM = Time in years
NMX = Maximum degree + 1
L = Control variable (Set > 0 to read in coefficients)
X = North Component of Field
Y = East Component
Z = Vertical Component (Positive Down)
F = Total Field

VARIABLES DESCRIBING INPUT COEFFICIENTS:

J ≠ 0 Coefficients derived using spherical earth approximation
J = 0 Coefficients derived using oblate earth

K = 0 Schmidt Normalized
K ≠ 0 Gauss Normalized

*Numbers at Branch Points are statement numbers

Figure 3

Table I

```
      SUBROUTINE FIELDG (DLAT,DLONG,ALT,TM,NMX,L,X,Y,Z,F)                    1
C     GEOMAGNETIC FIELD USING ANY SET OF COEFFICIENTS                        2
C     L POSITIVE READS A NEW SET OF COEFFICIENTS. FIRST CARD                 3
C     READ AS IDENTIFICATION IN COLUMNS 10-72.IF COL. 1 ZERO THEN            4
C     COEFFICIENTS ARE FOR OBLATE EARTH, IF NON ZERO THEN SPHERICAL          5
C     EARTH. IF COL. 2 ZERO  THEN SCHMIDT NORMALIZED, IF NON-ZERO THEN       6
C     GAUSS NORMALIZED. COLS. 4-9 CONTAIN EPOCH YEAR.                        7
C     DLAT,ALT,X AND Z ARE GEODETIC                                          8
C      REQUIRES SUBROUTINE FIELD(ST,CT,SPH,CPH,R,NMAX,BT,BP,BR,B)            9
      DIMENSION H(18,18),G(18,18),HT(18,18),GT(18,18),HTT(18,18),GTT(18,    10
     118),SHMIT(18,18),AID(11)                                              11
      COMMON /COEFFS/TG(18,18),TH(18,18)                                    12
      IF (SHMIT(1,1)+1.) 1,3,1                                              13
      A=6378.165                                                            14
      FLAT=1.-1./298.3                                                      15
      A2=A**2                                                               16
      A4=A**4                                                               17
      B2=(A*FLAT)**2                                                        18
      A2B2=A2*(1.-FLAT**2)                                                  19
      A4B4=A4*(1.-FLAT**4)                                                  20
      SHMIT(1,1)=-1.                                                        21
      DO 2 N=2,18                                                           22
      SHMIT(N,1)=SHMIT(N-1,1)*FLOAT(2*N-3)/FLOAT(N-1)                       23
      J=2                                                                   24
      DO 2 M=2,N                                                            25
      SHMIT(N,M)=SHMIT(N,M-1)*SQRT(FLOAT((N-M+1)*J)/FLOAT(N+M-2))           26
2     J=1                                                                   27
3     IF (L) 19,4,5                                                         28
4     IF (TM-TLAST) 17,19,17                                                29
5     READ (5,6) J,K,TZERO,(AID(I),I=1,11)                                  30
6     FORMAT (2I1,1X,F6.1,10A6,A3)                                          31
      WRITE (6,7) J,K,TZERO,(AID(I),I=1,11)                                 32
7     FORMAT (2I3,5X,6HEPOCH=,F7.1,5X10A6,A3)                               33
      MAXN=0                                                                34
      TEMP=0.                                                               35
8     READ (5,9) N,M,GNM,HNM,GTNM,HTNM,GTTNM,HTTNM                          36
9     FORMAT (2I3,6F11.4)                                                   37
      IF (N) 11,11,10                                                       38
10    MAXN=(MAXO(N,MAXN))                                                   39
      G(N,M)=GNM                                                            40
      H(N,M)=HNM                                                            41
      GT(N,M)=GTNM                                                          42
      HT(N,M)=HTNM                                                          43
      GTT(N,M)=GTTNM                                                        44
      HTT(N,M)=HTTNM                                                        45
      TEMP=AMAX1(TEMP,ABS(GTNM))                                            46
      GO TO 8                                                               47
11    WRITE (6,25) ((N,M,G(N,M),H(N,M),GT(N,M),HT(N,M),GTT(N,M),HTT(N,M)    48
     1,M=1,N),N=2,MAXN)                                                     49
      IF (TEMP) 13,12,13                                                    50
12    L=-L                                                                  51
      GO TO 14                                                              52
13    L=0                                                                   53
14    IF (K) 17,15,17                                                       54
15    DO 16 N=2,MAXN                                                        55
      DO 16 M=1,N                                                           56
      G(N,M)=G(N,M)*SHMIT(N,M)                                              57
      H(N,M)=H(N,M)*SHMIT(N,M)                                              58
      GT(N,M)=GT(N,M)*SHMIT(N,M)                                            59
      HT(N,M)=HT(N,M)*SHMIT(N,M)                                            60
      GTT(N,M)=GTT(N,M)*SHMIT(N,M)                                          61
```

Table I Cont.

```
16    HTT(N,M)=HTI(N,M)*SHMIT(N,M)                                          62
17    T=TM-TZERO                                                            63
      DO 18 N=2,MAXN                                                        64
      DO 18 M=1,N                                                           65
      TG(N,M)=G(N,M)+T*(GT(N,M)+GTT(N,M)*T)                                 66
18    TH(N,M)=H(N,M)+T*(HT(N,M)+HTT(N,M)*T)                                 67
      TLAST=TM                                                              68
19    SINLA=SIN(DLAT/57.2957795)                                            69
      RLONG=DLONG/57.2957795                                                70
      CPH=COS(RLONG)                                                        71
      SPH=SIN(RLONG)                                                        72
      IF (J) 20,21,20                                                       73
20    R=ALT+6371.2                                                          74
      CT=SINLA                                                              75
      GO TO 22                                                              76
21    SINLA2=SINLA**2                                                       77
      COSLA2=1.-SINLA2                                                      78
      DEN2=A2-A2B2*SINLA2                                                   79
      DEN=SQRT(DEN2)                                                        80
      FAC=(((ALT*DEN)+A2)/((ALT*DEN)+B2))**2                                81
      CT=SINLA/SQRT(FAC*COSLA2+SINLA2)                                      82
      R=SQRT(ALT*(ALT+2.*DEN)+(A4-A4B4*SINLA2)/DEN2)                        83
22    ST=SQRT(1.-CT**2)                                                     84
      NMAX=MIND(NMX,MAXN)                                                   85
      CALL FIELD (ST,CT,SPH,CPH,R,NMAX,BT,Y,BR,F)                           86
      IF (J) 23,24,23                                                       87
23    X=-BT                                                                 88
      Z=-BR                                                                 89
      RETURN                                                                90
C     TRANSFORMS FIELD TO GEODETIC DIRECTIONS                               91
24    SIND=SINLA*ST-SQRT(COSLA2)*CT                                         92
      COSD=SQRT(1.0-SIND**2)                                                93
      X=-BT*COSD-BR*SIND                                                    94
      Z=BT*SIND-BR*COSD                                                     95
25    FORMAT (6HO N  M6X1HG10X1HH9X2HGT9X2HHT9X3HGTT8X3HHTT//(2I3,6F11.4    96
     1))                                                                    97
      RETURN                                                                98
      END                                                                   99
```

Table II

```
      SUBROUTINE FIELD (ST,CT,SPH,CPH,R,NMAX,BT,BP,BR,B)                    1
      COMMON /COEFFS/G(18,18),H(18,18)                                      2
      DIMENSION P(18,18),DP(18,18),CONST(18,18),SP(18),CP(18),FN(18),FM(    3
     118)                                                                   4
      IF (P(1,1)-1.0) 1,3,1                                                 5
1     P(1,1)=1.                                                             6
      DP(1,1)=0.                                                            7
      SP(1)=0.                                                              8
      CP(1)=1.                                                              9
      DO 2 N=2,18                                                          10
      FN(N)=N                                                              11
      DO 2 M=1,N                                                           12
      FM(M)=M-1                                                            13
2     CONST(N,M)=FLOAT((N-2)**2-(M-1)**2)/FLOAT((2*N-3)*(2*N-5))           14
      CONST(2,2)=0.                                                        15
3     SP(2)=SPH                                                            16
      CP(2)=CPH                                                            17
      DO 4 M=3,NMAX                                                        18
      SP(M)=SP(2)*CP(M-1)+CP(2)*SP(M-1)                                    19
4     CP(M)=CP(2)*CP(M-1)-SP(2)*SP(M-1)                                    20
      AOR=6371.2/R                                                         21
      AR=AOR**2                                                            22
      BT=0.                                                                23
      BP=0.                                                                24
      BR=0.                                                                25
      DO 8 N=2,NMAX                                                        26
      AR=AOR*AR                                                            27
      DO 8 M=1,N                                                           28
      IF (N-M) 6,5,6                                                       29
5     P(N,N)=ST*P(N-1,N-1)                                                 30
      DP(N,N)=ST*DP(N-1,N-1)+CT*P(N-1,N-1)                                 31
      GO TO 7                                                              32
6     P(N,M)=CT*P(N-1,M)-CONST(N,M)*P(N-2,M)                               33
      DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)-CONST(N,M)*DP(N-2,M)                34
7     PAR=P(N,M)*AR                                                        35
      TEMP=G(N,M)*CP(M)+H(N,M)*SP(M)                                       36
      BT=BT+TEMP*DP(N,M)*AR                                                37
      BP=BP-(G(N,M)*SP(M)-H(N,M)*CP(M))*FM(M)*PAR                          38
8     BR=BR-TEMP*FN(N)*PAR                                                 39
      BP=BP/ST                                                             40
      B=SQRT(BT*BT+BP*BP+BR*BR)                                            41
      RETURN                                                               42
      END                                                                  43
```

Table III

```
C     PUNCHES FORTRAN TEXT FOR NON-SUBSCRIPTED FIELD PROGRAM
      PUNCH 1                                                              4
1     FORMAT (6X,50HSUBROUTINE FIELD (ST,CT,SPH,CPH,R,NMAX,BT,BP,BR,B))    5
      PUNCH 2                                                              6
2     FORMAT (6X,31HCOMMON/COEFFS/G(18,18),H(18,18))                       7
      PUNCH 3                                                              8
3     FORMAT (6X,11HAR=6371.2/R)                                           9
      PUNCH 4                                                             10
4     FORMAT (1HC,59X,4HN= 2)                                             11
      PUNCH 5                                                             12
5     FORMAT (6X,7HSP2=SPH)                                               13
      PUNCH 6                                                             14
6     FORMAT (6X,7HCP2=CPH)                                               15
      PUNCH 7                                                             16
7     FORMAT (6X,6HP21=CT)                                                17
      PUNCH 8                                                             18
8     FORMAT (6X,6HP22=ST)                                                19
      PUNCH 9                                                             20
9     FORMAT (6X,9HDP21=-P22)                                             21
      PUNCH 10                                                            22
10    FORMAT (6X,8HDP22=P21)                                              23
      PUNCH 11                                                            24
11    FORMAT (6X,12HAOR=AR*AR*AR)                                         25
      PUNCH 12                                                            26
12    FORMAT (6X,24HC2=G(2,2)*CP2+H(2,2)*SP2)                             27
      PUNCH 13                                                            28
13    FORMAT (6X,33HBR=-(AOR+AOR)*(G(2,1)*P21+C2*P22))                    29
      PUNCH 14                                                            30
14    FORMAT (6X,29HBT =AOR*(G(2,1)*DP21+C2*DP22))                        31
      PUNCH 15                                                            32
15    FORMAT (6X,35HBP =AOR*(H(2,2)*CP2-G(2,2)*SP2)*P22)                  33
      DO 57 N=3,18                                                        34
      NM=N-1                                                              35
      NM2=N-2                                                             36
      PUNCH 16,NM,N                                                       37
16    FORMAT (6X,8HIF(NMAX-,I2,6H) 1,1,,I2)                               38
      PUNCH 17,N                                                          39
17    FORMAT (1HC,59X,2HN=,I2)                                            40
      IF (MOD(N,2)) 18,21,18                                              41
18    NO2=NM/2+1                                                          42
      PUNCH 19,N,N,NO2,NO2,NO2                                            43
19    FORMAT (I2,4X,2HSP,I2,4H=(SP,I2,3H+SP,I2,4H)*CP,I2)                 44
      PUNCH 20,N,NO2,NO2,NO2,NO2                                          45
20    FORMAT (6X,2HCP,I2,4H=(CP,I2,3H+SP,I2,5H)*(CP,I2,3H-SP,I2,1H))      46
      GO TO 24                                                            47
21    PUNCH 22,N,N,NM,NM                                                  48
22    FORMAT (I2,4X,2HSP,I2,7H=SP2*CP,I2,7H+CP2*SP,I2)                    49
      PUNCH 23,N,NM,NM                                                    50
23    FORMAT (6X,2HCP,I2,7H=CP2*CP,I2,7H-SP2*SP,I2)                       51
24    IF (N-3) 32,25,32                                                   52
25    PUNCH 26                                                            53
26    FORMAT (6X,23HP31=P21*P21-0.333333333)                              54
      PUNCH 27                                                            55
27    FORMAT (6X,11HP32=P21*P22)                                          56
      PUNCH 28                                                            57
28    FORMAT (6X,11HP33=P22*P22)                                          58
      PUNCH 29                                                            59
29    FORMAT (6X,13HDP31=-P32-P32)                                        60
      PUNCH 30                                                            61
30    FORMAT (6X,16HDP32=P21*P21-P33)                                     62
      PUNCH 31                                                            63
```

Table III (Cont)

```
31    FORMAT (6X,10HDP33=-DP31)                                          64
      GO TO 43                                                           65
32    DO 40 M=1,NM                                                       66
      INUM=(N+M-3)*(N-M-1)                                               67
      IDEN=(N+N-3)*(N+N-5)                                               68
      IC1=(10000*INUM)/IDEN+10000                                        69
      INUM=MOD(10000*INUM,IDEN)                                          70
      IC2=(10000*INUM)/IDEN+10000                                        71
      PUNCH 33,N,M,NM,M                                                  72
33    FORMAT (6X,1HP,2I2,6H=P21*P,2I2)                                   73
      IF (M-NM) 34,36,34                                                 74
34    PUNCH 35,IC1,IC2,NM2,M                                             75
35    FORMAT (5X,4H$-0.,2I4,2H*P,2I2)                                    76
36    PUNCH 37,N,M,NM,M,NM,M                                             77
37    FORMAT (6X,2HDP,2I2,7H=P21*DP,2I2,7H+DP21*P,2I2)                   78
      IF (M-NM) 38,40,38                                                 79
38    PUNCH 39,IC1,IC2,NM2,M                                             80
39    FORMAT (5X,4H$-0.,2I4,3H*DP,2I2)                                   81
40    CONTINUE                                                           82
      PUNCH 41,N,N,NM,NM                                                 83
41    FORMAT (6X,1HP,2I2,6H=P22*P,2I2)                                   84
      PUNCH 42,N,N,NM,N,NM                                               85
42    FORMAT (6X,2HDP,2I2,1H=,I2,4H.0*P,2I2)                             86
43    PUNCH 44                                                           87
44    FORMAT (6X,10HAOR=AOR*AR)                                          88
      DO 46 M=2,N                                                        89
      PUNCH 45,M,N,M,M,N,M,M                                             90
45    FORMAT (6X,1HC,I2,3H=G(,I2,1H,,I2,4H)*CP,I2,3H+H(,I2,1H,,I2,4H)*SP 91
     1,I2)                                                               92
46    CONTINUE                                                           93
      PUNCH 47,N,N,N                                                     94
47    FORMAT (6X,8HBR =BR -,I2,10H.0*AOR*(G(,I2,5H,1)*P,I2,1H1)          95
      DO 49 M=2,N                                                        96
      PUNCH 48,M,N,M                                                     97
48    FORMAT (5X,3H$+C,I2,2H*P,2I2)                                      98
49    CONTINUE                                                           99
      PUNCH 50                                                          100
50    FORMAT (5X,2H$))                                                  101
      PUNCH 51,N,N                                                      102
51    FORMAT (6X,15HBT =BT +AOR*(G(,I2,6H,1)*DP,I2,1H1)                 103
      DO 53 M=2,N                                                       104
      PUNCH 52,M,N,M                                                    105
52    FORMAT (5X,3H$+C,I2,3H*DP,2I2)                                    106
53    CONTINUE                                                          107
      PUNCH 50                                                          108
      PUNCH 54,N,N,N                                                    109
54    FORMAT (6X,16HBP =BP -AOR*((G(,I2,10H,2)*SP2-H(,I2,10H,2)*CP2)*P,I 110
     12,1H2)                                                            111
      DO 56 M=3,N                                                       112
      MM=M-1                                                            113
      PUNCH 55,MM,N,M,M,N,M,M,N,M                                       114
55    FORMAT (5X,2H$+,I2,6H.0*(G(,I2,1H,,I2,4H)*SP,I2,3H-H(,I2,1H,,I2,4H 115
     1)*CP,I2,3H)*P,2I2)                                                116
56    CONTINUE                                                          117
      PUNCH 50                                                          118
57    CONTINUE                                                          119
      PUNCH 58                                                          120
58    FORMAT (1H1,5X9HBP=BP/P22)                                        121
      PUNCH 59                                                          122
59    FORMAT (6X,25HB=SQRT(BR*BR+BT*BT+BP*BP))                          123
      PUNCH 60                                                          124
```

Table III (Cont)

```
60      FORMAT (6X,6HRETURN)                                    125
        PUNCH 61                                                126
61      FORMAT (6X,3HEND)                                       127
        RETURN                                                  128
        END                                                     129
```

Table IV

```
      SUBROUTINE FIELD(ST,CT,SPH,CPH,R,NMAX,BT,BP,BR,B)                    1
      COMMON/COEFFS/G(18,18),H(18,18)                                      2
      AR=6371.2/R                                                          3
C                                                        N= 2               4
1     SP2=SPH                                                              5
      CP2=CPH                                                              6
      P21=CT                                                               7
      P22=ST                                                               8
      DP21=-P22                                                            9
      DP22=P21                                                            10
      AOR=AR*AR*AR                                                        11
      C2=G(2,2)*CP2+H(2,2)*SP2                                            12
      BR=-(AOR+AOR)*(G(2,1)*P21+C2*P22)                                   13
      BT=AOR*(G(2,1)*DP21+C2*DP22)                                        14
      BP=AOR*(H(2,2)*CP2-G(2,2)*SP2)*P22                                  15
      IF(NMAX- 2) 18,18,2                                                 16
C                                                        N= 3               17
2     SP3=(SP2+SP2)*CP2                                                   18
      CP3=(CP2+SP2)*(CP2-SP2)                                             19
      P31=P21*P21-0.333333333                                             20
      P32=P21*P22                                                         21
      P33=P22*P22                                                         22
      DP31=-P32-P32                                                       23
      DP32=P21*P21-P33                                                    24
      DP33=-DP31                                                          25
      AOR=AOR*AR                                                          26
      C2=G(3,2)*CP2+H(3,2)*SP2                                            27
      C3=G(3,3)*CP3+H(3,3)*SP3                                            28
      BR=BR-3.0*AOR*(G(3,1)*P31+C2*P32+C3*P33)                            29
      BT=BT+AOR*(G(3,1)*DP31+C2*DP32+C3*DP33)                             30
      BP=BP-AOR*((G(3,2)*SP2-H(3,2)*CP2)*P32+2.0*(G(3,3)*SP3-H(3,3)*CP3)  31
     1*P33)                                                               32
      IF(NMAX- 3) 18,18,3                                                 33
C                                                        N= 4               34
3     SP4=SP2*CP3+CP2*SP3                                                 35
      CP4=CP2*CP3-SP2*SP3                                                 36
      P41=P21*P31-0.26666666*P21                                          37
      DP41=P21*DP31+DP21*P31-0.26666666*DP21                              38
      P42=P21*P32-0.20000000*P22                                          39
      DP42=P21*DP32+DP21*P32-0.20000000*DP22                              40
      P43=P21*P33                                                         41
      DP43=P21*DP33+DP21*P33                                              42
      P44=P22*P33                                                         43
      DP44=3.0*P43                                                        44
      AOR=AOR*AR                                                          45
      C2=G(4,2)*CP2+H(4,2)*SP2                                            46
      C3=G(4,3)*CP3+H(4,3)*SP3                                            47
      C4=G(4,4)*CP4+H(4,4)*SP4                                            48
      BR=BR-4.0*AOR*(G(4,1)*P41+C2*P42+C3*P43+C4*P44)                     49
      BT=BT+AOR*(G(4,1)*DP41+C2*DP42+C3*DP43+C4*DP44)                     50
      BP=BP-AOR*((G(4,2)*SP2-H(4,2)*CP2)*P42+2.0*(G(4,3)*SP3-H(4,3)*CP3)  51
     1*P43+3.0*(G(4,4)*SP4-H(4,4)*CP4)*P44)                               52
      IF(NMAX- 4) 18,18,4                                                 53
C                                                        N= 5               54
4     SP5=(SP3+SP3)*CP3                                                   55
      CP5=(CP3+SP3)*(CP3-SP3)                                             56
      P51=P21*P41-0.25714285*P31                                          57
      DP51=P21*DP41+DP21*P41-0.25714285*DP31                              58
      P52=P21*P42-0.22857142*P32                                          59
      DP52=P21*DP42+DP21*P42-0.22857142*DP32                              60
      P53=P21*P43-0.14285714*P33                                          61
```

Table IV (Cont)

```
      DP53=P21*DP43+DP21*P43-0.14285714*DP33                                    62
      P54=P21*P44                                                               63
      DP54=P21*DP44+DP21*P44                                                    64
      P55=P22*P44                                                               65
      DP55=4.0*P54                                                              66
      AOR=AOR*AR                                                                67
      C2=G(5,2)*CP2+H(5,2)*SP2                                                  68
      C3=G(5,3)*CP3+H(5,3)*SP3                                                  69
      C4=G(5,4)*CP4+H(5,4)*SP4                                                  70
      C5=G(5,5)*CP5+H(5,5)*SP5                                                  71
      BR=BR-5.0*AOR*(G(5,1)*P51+C2*P52+C3*P53+C4*P54+C5*P55)                    72
      BT=BT+AOR*(G(5,1)*DP51+C2*DP52+C3*DP53+C4*DP54+C5*DP55)                   73
      BP=BP-AOR*((G(5,2)*SP2-H(5,2)*CP2)*P52+2.0*(G(5,3)*SP3-H(5,3)*CP3)       74
     1*P53+3.0*(G(5,4)*SP4-H(5,4)*CP4)*P54+4.0*(G(5,5)*SP5-H(5,5)*CP5)*P       75
     255)                                                                       76
      IF(NMAX- 5) 18,18,5                                                       77
C                                                          N= 6                 78
5     SP6=SP2*CP5+CP2*SP5                                                       79
      CP6=CP2*CP5-SP2*SP5                                                       80
      P61=P21*P51-0.25396825*P41                                                81
      DP61=P21*DP51+DP21*P51-0.25396825*DP41                                    82
      P62=P21*P52-0.23809523*P42                                                83
      DP62=P21*DP52+DP21*P52-0.23809523*DP42                                    84
      P63=P21*P53-0.19047619*P43                                                85
      DP63=P21*DP53+DP21*P53-0.19047619*DP43                                    86
      P64=P21*P54-0.11111111*P44                                                87
      DP64=P21*DP54+DP21*P54-0.11111111*DP44                                    88
      P65=P21*P55                                                               89
      DP65=P21*DP55+DP21*P55                                                    90
      P66=P22*P55                                                               91
      DP66=5.0*P65                                                              92
      AOR=AOR*AR                                                                93
      C2=G(6,2)*CP2+H(6,2)*SP2                                                  94
      C3=G(6,3)*CP3+H(6,3)*SP3                                                  95
      C4=G(6,4)*CP4+H(6,4)*SP4                                                  96
      C5=G(6,5)*CP5+H(6,5)*SP5                                                  97
      C6=G(6,6)*CP6+H(6,6)*SP6                                                  98
      BR=BR-6.0*AOR*(G(6,1)*P61+C2*P62+C3*P63+C4*P64+C5*P65+C6*P66)             99
      BT=BT+AOR*(G(6,1)*DP61+C2*DP62+C3*DP63+C4*DP64+C5*DP65+C6*DP66)          100
      BP=BP-AOR*((G(6,2)*SP2-H(6,2)*CP2)*P62+2.0*(G(6,3)*SP3-H(6,3)*CP3)       101
     1*P63+3.0*(G(6,4)*SP4-H(6,4)*CP4)*P64+4.0*(G(6,5)*SP5-H(6,5)*CP5)*P       102
     265+5.0*(G(6,6)*SP6-H(6,6)*CP6)*P66)                                      103
      IF(NMAX- 6) 18,18,6                                                      104
C                                                          N= 7                105
6     SP7=(SP4+SP4)*CP4                                                        106
      CP7=(CP4+SP4)*(CP4-SP4)                                                  107
      P71=P21*P61-0.25252525*P51                                               108
      DP71=P21*DP61+DP21*P61-0.25252525*DP51                                   109
      P72=P21*P62-0.24242424*P52                                               110
      DP72=P21*DP62+DP21*P62-0.24242424*DP52                                   111
      P73=P21*P63-0.21212121*P53                                               112
      DP73=P21*DP63+DP21*P63-0.21212121*DP53                                   113
      P74=P21*P64-0.16161616*P54                                               114
      DP74=P21*DP64+DP21*P64-0.16161616*DP54                                   115
      P75=P21*P65-0.09090909*P55                                               116
      DP75=P21*DP65+DP21*P65-0.09090909*DP55                                   117
      P76=P21*P66                                                              118
      DP76=P21*DP66+DP21*P66                                                   119
      P77=P22*P66                                                              120
      DP77=6.0*P76                                                             121
      AOR=AOR*AR                                                               122
```

Table IV (Cont)

```
      C2=G(7,2)*CP2+H(7,2)*SP2                                                  123
      C3=G(7,3)*CP3+H(7,3)*SP3                                                  124
      C4=G(7,4)*CP4+H(7,4)*SP4                                                  125
      C5=G(7,5)*CP5+H(7,5)*SP5                                                  126
      C6=G(7,6)*CP6+H(7,6)*SP6                                                  127
      C7=G(7,7)*CP7+H(7,7)*SP7                                                  128
      BR=BR-7.0*AOR*(G(7,1)*P71+C2*P72+C3*P73+C4*P74+C5*P75+C6*P76+C7*P7        129
     17)                                                                        130
      BT=BT+AOR*(G(7,1)*DP71+C2*DP72+C3*DP73+C4*DP74+C5*DP75+C6*DP76+C7*        131
     1DP77)                                                                     132
      BP=BP-AOR*((G(7,2)*SP2-H(7,2)*CP2)*P72+2.0*(G(7,3)*SP3-H(7,3)*CP3)        133
     1*P73+3.0*(G(7,4)*SP4-H(7,4)*CP4)*P74+4.0*(G(7,5)*SP5-H(7,5)*CP5)*P       134
     275+5.0*(G(7,6)*SP6-H(7,6)*CP6)*P76+6.0*(G(7,7)*SP7-H(7,7)*CP7)*P77       135
     3)                                                                         136
      IF(NMAX- 7) 18,18,7                                                       137
C                                                          N= 8                 138
7     SP8=SP2*CP7+CP2*SP7                                                       139
      CP8=CP2*CP7-SP2*SP7                                                       140
      P81=P21*P71-0.25174825*P61                                                141
      DP81=P21*DP71+DP21*P71-0.25174825*DP61                                    142
      P82=P21*P72-0.24475524*P62                                                143
      DP82=P21*DP72+DP21*P72-0.24475524*DP62                                    144
      P83=P21*P73-0.22377622*P63                                                145
      DP83=P21*DP73+DP21*P73-0.22377622*DP63                                    146
      P84=P21*P74-0.18881118*P64                                                147
      DP84=P21*DP74+DP21*P74-0.18881118*DP64                                    148
      P85=P21*P75-0.13986013*P65                                                149
      DP85=P21*DP75+DP21*P75-0.13986013*DP65                                    150
      P86=P21*P76-0.07692307*P66                                                151
      DP86=P21*DP76+DP21*P76-0.07692307*DP66                                    152
      P87=P21*P77                                                               153
      DP87=P21*DP77+DP21*P77                                                    154
      P88=P22*P77                                                               155
      DP88=7.0*P87                                                              156
      AOR=AOR*AR                                                                157
      C2=G(8,2)*CP2+H(8,2)*SP2                                                  158
      C3=G(8,3)*CP3+H(8,3)*SP3                                                  159
      C4=G(8,4)*CP4+H(8,4)*SP4                                                  160
      C5=G(8,5)*CP5+H(8,5)*SP5                                                  161
      C6=G(8,6)*CP6+H(8,6)*SP6                                                  162
      C7=G(8,7)*CP7+H(8,7)*SP7                                                  163
      C8=G(8,8)*CP8+H(8,8)*SP8                                                  164
      BR=BR-8.0*AOR*(G(8,1)*P81+C2*P82+C3*P83+C4*P84+C5*P85+C6*P86+C7*P8        165
     17+C8*P88)                                                                 166
      BT=BT+AOR*(G(8,1)*DP81+C2*DP82+C3*DP83+C4*DP84+C5*DP85+C6*DP86+C7*        167
     1DP87+C8*DP88)                                                             168
      BP=BP-AOR*((G(8,2)*SP2-H(8,2)*CP2)*P82+2.0*(G(8,3)*SP3-H(8,3)*CP3)        169
     1*P83+3.0*(G(8,4)*SP4-H(8,4)*CP4)*P84+4.0*(G(8,5)*SP5-H(8,5)*CP5)*P       170
     285+5.0*(G(8,6)*SP6-H(8,6)*CP6)*P86+6.0*(G(8,7)*SP7-H(8,7)*CP7)*P87       171
     3+7.0*(G(8,8)*SP8-H(8,8)*CP8)*P88)                                         172
      IF(NMAX- 8) 18,18,8                                                       173
C                                                          N= 9                 174
8     SP9=(SP5+SP5)*CP5                                                         175
      CP9=(CP5+SP5)*(CP5-SP5)                                                   176
      P91=P21*P81-0.25128205*P71                                                177
      DP91=P21*DP81+DP21*P81-0.25128205*DP71                                    178
      P92=P21*P82-0.24615384*P72                                                179
      DP92=P21*DP82+DP21*P82-0.24615384*DP72                                    180
      P93=P21*P83-0.23076923*P73                                                181
      DP93=P21*DP83+DP21*P83-0.23076923*DP73                                    182
      P94=P21*P84-0.20512820*P74                                                183
```

Table IV (Cont)

```
      DP94=P21*DP84+DP21*P84-0.20512820*DP74                                  184
      P95=P21*P85-0.16923076*P75                                              185
      DP95=P21*DP85+DP21*P85-0.16923076*DP75                                  186
      P96=P21*P86-0.12307692*P76                                              187
      DP96=P21*DP86+DP21*P86-0.12307692*DP76                                  188
      P97=P21*P87-0.06666666*P77                                              189
      DP97=P21*DP87+DP21*P87-0.06666666*DP77                                  190
      P98=P21*P88                                                             191
      DP98=P21*DP88+DP21*P88                                                  192
      P99=P22*P88                                                             193
      DP99=8.0*P98                                                            194
      AOR=AOR*AR                                                              195
      C2=G(9,2)*CP2+H(9,2)*SP2                                                196
      C3=G(9,3)*CP3+H(9,3)*SP3                                                197
      C4=G(9,4)*CP4+H(9,4)*SP4                                                198
      C5=G(9,5)*CP5+H(9,5)*SP5                                                199
      C6=G(9,6)*CP6+H(9,6)*SP6                                                200
      C7=G(9,7)*CP7+H(9,7)*SP7                                                201
      C8=G(9,8)*CP8+H(9,8)*SP8                                                202
      C9=G(9,9)*CP9+H(9,9)*SP9                                                203
      BR=BR-9.0*AOR*(G(9,1)*P91+C2*P92+C3*P93+C4*P94+C5*P95+C6*P96+C7*P9      204
     17+C8*P98+C9*P99)                                                        205
      BT=BT+AOR*(G(9,1)*DP91+C2*DP92+C3*DP93+C4*DP94+C5*DP95+C6*DP96+C7*      206
     1DP97+C8*DP98+C9*DP99)                                                   207
      BP=BP-AOR*((G(9,2)*SP2-H(9,2)*CP2)*P92+2.0*(G(9,3)*SP3-H(9,3)*CP3)      208
     1*P93+3.0*(G(9,4)*SP4-H(9,4)*CP4)*P94+4.0*(G(9,5)*SP5-H(9,5)*CP5)*P      209
     295+5.0*(G(9,6)*SP6-H(9,6)*CP6)*P96+6.0*(G(9,7)*SP7-H(9,7)*CP7)*P97      210
     3+7.0*(G(9,8)*SP8-H(9,8)*CP8)*P98+8.0*(G(9,9)*SP9-H(9,9)*CP9)*P99)       211
      IF(NMAX- 9) 18,18,9                                                     212
C                                                          N=10                213
9     SP10=SP2*CP9+CP2*SP9                                                    214
      CP10=CP2*CP9-SP2*SP9                                                    215
      P101=P21*P91-0.25098039*P81                                             216
      DP101=P21*DP91+DP21*P91-0.25098039*DP81                                 217
      P102=P21*P92-0.24705882*P82                                             218
      DP102=P21*DP92+DP21*P92-0.24705882*DP82                                 219
      P103=P21*P93-0.23529411*P83                                             220
      DP103=P21*DP93+DP21*P93-0.23529411*DP83                                 221
      P104=P21*P94-0.21568627*P84                                             222
      DP104=P21*DP94+DP21*P94-0.21568627*DP84                                 223
      P105=P21*P95-0.18823529*P85                                             224
      DP105=P21*DP95+DP21*P95-0.18823529*DP85                                 225
      P106=P21*P96-0.15294117*P86                                             226
      DP106=P21*DP96+DP21*P96-0.15294117*DP86                                 227
      P107=P21*P97-0.10980392*P87                                             228
      DP107=P21*DP97+DP21*P97-0.10980392*DP87                                 229
      P108=P21*P98-0.05882352*P88                                             230
      DP108=P21*DP98+DP21*P98-0.05882352*DP88                                 231
      P109=P21*P99                                                            232
      DP109=P21*DP99+DP21*P99                                                 233
      P1010=P22*P99                                                           234
      DP1010=9.0*P109                                                         235
      AOR=AOR*AR                                                              236
      C2=G(10,2)*CP2+H(10,2)*SP2                                              237
      C3=G(10,3)*CP3+H(10,3)*SP3                                              238
      C4=G(10,4)*CP4+H(10,4)*SP4                                              239
      C5=G(10,5)*CP5+H(10,5)*SP5                                              240
      C6=G(10,6)*CP6+H(10,6)*SP6                                              241
      C7=G(10,7)*CP7+H(10,7)*SP7                                              242
      C8=G(10,8)*CP8+H(10,8)*SP8                                              243
      C9=G(10,9)*CP9+H(10,9)*SP9                                              244
```

Table IV (Cont)

```
      C10=G(10,10)*CP10+H(10,10)*SP10                                    245
      BR=BR-10.0*AOR*(G(10,1)*P101+C2*P102+C3*P103+C4*P104+C5*P105+C6*P1  246
     106+C7*P107+C8*P108+C9*P109+C10*P1010)                              247
      BT=BT+AOR*(G(10,1)*DP101+C2*DP102+C3*DP103+C4*DP104+C5*DP105+C6*DP 248
     1106+C7*DP107+C8*DP108+C9*DP109+C10*DP1010)                         249
      BP=BP-AOR*((G(10,2)*SP2-H(10,2)*CP2)*P102+2.0*(G(10,3)*SP3-H(10,3) 250
     1*CP3)*P103+3.0*(G(10,4)*SP4-H(10,4)*CP4)*P104+4.0*(G(10,5)*SP5-H(1 251
     20,5)*CP5)*P105+5.0*(G(10,6)*SP6-H(10,6)*CP6)*P106+6.0*(G(10,7)*SP7 252
     3-H(10,7)*CP7)*P107+7.0*(G(10,8)*SP8-H(10,8)*CP8)*P108+8.0*(G(10,9) 253
     4*SP9-H(10,9)*CP9)*P109+9.0*(G(10,10)*SP10-H(10,10)*CP10)*P1010)    254
      IF(NMAX-10) 18,18,10                                               255
C                                                          N=11          256
10    SP11=(SP6+SP6)*CP6                                                 257
      CP11=(CP6+SP6)*(CP6-SP6)                                           258
      P111=P21*P101-0.25077399*P91                                       259
      DP111=P21*DP101+DP21*P101-0.25077399*DP91                          260
      P112=P21*P102-0.24767801*P92                                       261
      DP112=P21*DP102+DP21*P102-0.24767801*DP92                          262
      P113=P21*P103-0.23839009*P93                                       263
      DP113=P21*DP103+DP21*P103-0.23839009*DP93                          264
      P114=P21*P104-0.22291021*P94                                       265
      DP114=P21*DP104+DP21*P104-0.22291021*DP94                          266
      P115=P21*P105-0.20123839*P95                                       267
      DP115=P21*DP105+DP21*P105-0.20123839*DP95                          268
      P116=P21*P106-0.17337461*P96                                       269
      DP116=P21*DP106+DP21*P106-0.17337461*DP96                          270
      P117=P21*P107-0.13931888*P97                                       271
      DP117=P21*DP107+DP21*P107-0.13931888*DP97                          272
      P118=P21*P108-0.09907120*P98                                       273
      DP118=P21*DP108+DP21*P108-0.09907120*DP98                          274
      P119=P21*P109-0.05263157*P99                                       275
      DP119=P21*DP109+DP21*P109-0.05263157*DP99                          276
      P1110=P21*P1010                                                    277
      DP1110=P21*DP1010+DP21*P1010                                       278
      P1111=P22*P1010                                                    279
      DP1111=10.0*P1110                                                  280
      AOR=AOR*AR                                                         281
      C2=G(11,2)*CP2+H(11,2)*SP2                                         282
      C3=G(11,3)*CP3+H(11,3)*SP3                                         283
      C4=G(11,4)*CP4+H(11,4)*SP4                                         284
      C5=G(11,5)*CP5+H(11,5)*SP5                                         285
      C6=G(11,6)*CP6+H(11,6)*SP6                                         286
      C7=G(11,7)*CP7+H(11,7)*SP7                                         287
      C8=G(11,8)*CP8+H(11,8)*SP8                                         288
      C9=G(11,9)*CP9+H(11,9)*SP9                                         289
      C10=G(11,10)*CP10+H(11,10)*SP10                                    290
      C11=G(11,11)*CP11+H(11,11)*SP11                                    291
      BR=BR-11.0*AOR*(G(11,1)*P111+C2*P112+C3*P113+C4*P114+C5*P115+C6*P1 292
     116+C7*P117+C8*P118+C9*P119+C10*P1110+C11*P1111)                    293
      BT=BT+AOR*(G(11,1)*DP111+C2*DP112+C3*DP113+C4*DP114+C5*DP115+C6*DP 294
     1116+C7*DP117+C8*DP118+C9*DP119+C10*DP1110+C11*DP1111)              295
      BP=BP-AOR*((G(11,2)*SP2-H(11,2)*CP2)*P112+2.0*(G(11,3)*SP3-H(11,3) 296
     1*CP3)*P113+3.0*(G(11,4)*SP4-H(11,4)*CP4)*P114+4.0*(G(11,5)*SP5-H(1 297
     21,5)*CP5)*P115+5.0*(G(11,6)*SP6-H(11,6)*CP6)*P116+6.0*(G(11,7)*SP7 298
     3-H(11,7)*CP7)*P117+7.0*(G(11,8)*SP8-H(11,8)*CP8)*P118+8.0*(G(11,9) 299
     4*SP9-H(11,9)*CP9)*P119+9.0*(G(11,10)*SP10-H(11,10)*CP10)*P1110+10. 300
     50*(G(11,11)*SP11-H(11,11)*CP11)*P1111)                             301
      IF(NMAX-11) 18,18,11                                               302
C                                                          N=12          303
11    SP12=SP2*CP11+CP2*SP11                                             304
      CP12=CP2*CP11-SP2*SP11                                             305
```

Table IV (Cont)

```
      P121=P21*P111-0.25062656*P101                                          306
      DP121=P21*DP111+DP21*P111-0.25062656*DP101                             307
      P122=P21*P112-0.24812030*P102                                          308
      DP122=P21*DP112+DP21*P112-0.24812030*DP102                             309
      P123=P21*P113-0.24060150*P103                                          310
      DP123=P21*DP113+DP21*P113-0.24060150*DP103                             311
      P124=P21*P114-0.22807017*P104                                          312
      DP124=P21*DP114+DP21*P114-0.22807017*DP104                             313
      P125=P21*P115-0.21052631*P105                                          314
      DP125=P21*DP115+DP21*P115-0.21052631*DP105                             315
      P126=P21*P116-0.18796992*P106                                          316
      DP126=P21*DP116+DP21*P116-0.18796992*DP106                             317
      P127=P21*P117-0.16040100*P107                                          318
      DP127=P21*DP117+DP21*P117-0.16040100*DP107                             319
      P128=P21*P118-0.12781954*P108                                          320
      DP128=P21*DP118+DP21*P118-0.12781954*DP108                             321
      P129=P21*P119-0.09022556*P109                                          322
      DP129=P21*DP119+DP21*P119-0.09022556*DP109                             323
      P1210=P21*P1110-0.04761904*P1010                                       324
      DP1210=P21*DP1110+DP21*P1110-0.04761904*DP1010                         325
      P1211=P21*P1111                                                        326
      DP1211=P21*DP1111+DP21*P1111                                           327
      P1212=P22*P1111                                                        328
      DP1212=11.0*P1211                                                      329
      AOR=AOR*AR                                                             330
      C2=G(12,2)*CP2+H(12,2)*SP2                                             331
      C3=G(12,3)*CP3+H(12,3)*SP3                                             332
      C4=G(12,4)*CP4+H(12,4)*SP4                                             333
      C5=G(12,5)*CP5+H(12,5)*SP5                                             334
      C6=G(12,6)*CP6+H(12,6)*SP6                                             335
      C7=G(12,7)*CP7+H(12,7)*SP7                                             336
      C8=G(12,8)*CP8+H(12,8)*SP8                                             337
      C9=G(12,9)*CP9+H(12,9)*SP9                                             338
      C10=G(12,10)*CP10+H(12,10)*SP10                                        339
      C11=G(12,11)*CP11+H(12,11)*SP11                                        340
      C12=G(12,12)*CP12+H(12,12)*SP12                                        341
      BR=BR-12.0*AOR*(G(12,1)*P121+C2*P122+C3*P123+C4*P124+C5*P125+C6*P1     342
     1126+C7*P127+C8*P128+C9*P129+C10*P1210+C11*P1211+C12*P1212)             343
      BT=BT+AOR*(G(12,1)*DP121+C2*DP122+C3*DP123+C4*DP124+C5*DP125+C6*DP     344
     1126+C7*DP127+C8*DP128+C9*DP129+C10*DP1210+C11*DP1211+C12*DP1212)       345
      BP=BP-AOR*((G(12,2)*SP2-H(12,2)*CP2)*P122+2.0*(G(12,3)*SP3-H(12,3)     346
     1*CP3)*P123+3.0*(G(12,4)*SP4-H(12,4)*CP4)*P124+4.0*(G(12,5)*SP5-H(1     347
     22,5)*CP5)*P125+5.0*(G(12,6)*SP6-H(12,6)*CP6)*P126+6.0*(G(12,7)*SP7     348
     3-H(12,7)*CP7)*P127+7.0*(G(12,8)*SP8-H(12,8)*CP8)*P128+8.0*(G(12,9)     349
     4*SP9-H(12,9)*CP9)*P129+9.0*(G(12,10)*SP10-H(12,10)*CP10)*P1210+10.     350
     50*(G(12,11)*SP11-H(12,11)*CP11)*P1211+11.0*(G(12,12)*SP12-H(12,12)     351
     6*CP12)*P1212)                                                          352
      IF(NMAX-12) 18,18,12                                                   353
C                                                          N=13               354
12    SP13=(SP7+SP7)*CP7                                                     355
      CP13=(CP7+SP7)*(CP7-SP7)                                               356
      P131=P21*P121-0.25051759*P111                                          357
      DP131=P21*DP121+DP21*P121-0.25051759*DP111                             358
      P132=P21*P122-0.24844720*P112                                          359
      DP132=P21*DP122+DP21*P122-0.24844720*DP112                             360
      P133=P21*P123-0.24223602*P113                                          361
      DP133=P21*DP123+DP21*P123-0.24223602*DP113                             362
      P134=P21*P124-0.23188405*P114                                          363
      DP134=P21*DP124+DP21*P124-0.23188405*DP114                             364
      P135=P21*P125-0.21739130*P115                                          365
      DP135=P21*DP125+DP21*P125-0.21739130*DP115                             366
```

Table IV (Cont)

```
      P136=P21*P126-0.19875776*P116                                          367
      DP136=P21*DP126+DP21*P126-0.19875776*DP116                             368
      P137=P21*P127-0.17598343*P117                                          369
      DP137=P21*DP127+DP21*P127-0.17598343*DP117                             370
      P138=P21*P128-0.14906832*P118                                          371
      DP138=P21*DP128+DP21*P128-0.14906832*DP118                             372
      P139=P21*P129-0.11801242*P119                                          373
      DP139=P21*DP129+DP21*P129-0.11801242*DP119                             374
      P1310=P21*P1210-0.08281573*P1110                                       375
      DP1310=P21*DP1210+DP21*P1210-0.08281573*DP1110                         376
      P1311=P21*P1211-0.04347826*P1111                                       377
      DP1311=P21*DP1211+DP21*P1211-0.04347826*DP1111                         378
      P1312=P21*P1212                                                        379
      DP1312=P21*DP1212+DP21*P1212                                           380
      P1313=P22*P1212                                                        381
      DP1313=12.0*P1312                                                      382
      AOR=AOR*AR                                                             383
      C2=G(13,2)*CP2+H(13,2)*SP2                                             384
      C3=G(13,3)*CP3+H(13,3)*SP3                                             385
      C4=G(13,4)*CP4+H(13,4)*SP4                                             386
      C5=G(13,5)*CP5+H(13,5)*SP5                                             387
      C6=G(13,6)*CP6+H(13,6)*SP6                                             388
      C7=G(13,7)*CP7+H(13,7)*SP7                                             389
      C8=G(13,8)*CP8+H(13,8)*SP8                                             390
      C9=G(13,9)*CP9+H(13,9)*SP9                                             391
      C10=G(13,10)*CP10+H(13,10)*SP10                                        392
      C11=G(13,11)*CP11+H(13,11)*SP11                                        393
      C12=G(13,12)*CP12+H(13,12)*SP12                                        394
      C13=G(13,13)*CP13+H(13,13)*SP13                                        395
      BR=BR-13.0*AOR*(G(13,1)*P131+C2*P132+C3*P133+C4*P134+C5*P135+C6*P1     396
     136+C7*P137+C8*P138+C9*P139+C10*P1310+C11*P1311+C12*P1312+C13*P1313     397
     2)                                                                      398
      BT=BT+AOR*(G(13,1)*DP131+C2*DP132+C3*DP133+C4*DP134+C5*DP135+C6*DP     399
     1136+C7*DP137+C8*DP138+C9*DP139+C10*DP1310+C11*DP1311+C12*DP1312+C1     400
     23*DP1313)                                                              401
      BP=BP-AOR*((G(13,2)*SP2-H(13,2)*CP2)*P132+2.0*(G(13,3)*SP3-H(13,3)     402
     1*CP3)*P133+3.0*(G(13,4)*SP4-H(13,4)*CP4)*P134+4.0*(G(13,5)*SP5-H(1     403
     23,5)*CP5)*P135+5.0*(G(13,6)*SP6-H(13,6)*CP6)*P136+6.0*(G(13,7)*SP7     404
     3-H(13,7)*CP7)*P137+7.0*(G(13,8)*SP8-H(13,8)*CP8)*P138+8.0*(G(13,9)     405
     4*SP9-H(13,9)*CP9)*P139+9.0*(G(13,10)*SP10-H(13,10)*CP10)*P1310+10.     406
     50*(G(13,11)*SP11-H(13,11)*CP11)*P1311+11.0*(G(13,12)*SP12-H(13,12)     407
     6*CP12)*P1312+12.0*(G(13,13)*SP13-H(13,13)*CP13)*P1313)                 408
      IF(NMAX-13) 18,18,13                                                   409
C                                                        N=14                410
13    SP14=SP2*CP13+CP2*SP13                                                 411
      CP14=CP2*CP13-SP2*SP13                                                 412
      P141=P21*P131-0.25043478*P121                                          413
      DP141=P21*DP131+DP21*P131-0.25043478*DP121                             414
      P142=P21*P132-0.24869565*P122                                          415
      DP142=P21*DP132+DP21*P132-0.24869565*DP122                             416
      P143=P21*P133-0.24347826*P123                                          417
      DP143=P21*DP133+DP21*P133-0.24347826*DP123                             418
      P144=P21*P134-0.23478260*P124                                          419
      DP144=P21*DP134+DP21*P134-0.23478260*DP124                             420
      P145=P21*P135-0.22260869*P125                                          421
      DP145=P21*DP135+DP21*P135-0.22260869*DP125                             422
      P146=P21*P136-0.20695652*P126                                          423
      DP146=P21*DP136+DP21*P136-0.20695652*DP126                             424
      P147=P21*P137-0.18782608*P127                                          425
      DP147=P21*DP137+DP21*P137-0.18782608*DP127                             426
      P148=P21*P138-0.16521739*P128                                          427
```

Table IV (Cont)

```
      DP148=P21*DP138+DP21*P138-0.16521739*DP128                                428
      P149=P21*P139-0.13913043*P129                                             429
      DP149=P21*DP139+DP21*P139-0.13913043*DP129                                430
      P1410=P21*P1310-0.10956521*P1210                                          431
      DP1410=P21*DP1310+DP21*P1310-0.10956521*DP1210                            432
      P1411=P21*P1311-0.07652173*P1211                                          433
      DP1411=P21*DP1311+DP21*P1311-0.07652173*DP1211                            434
      P1412=P21*P1312-0.04000000*P1212                                          435
      DP1412=P21*DP1312+DP21*P1312-0.04000000*DP1212                            436
      P1413=P21*P1313                                                           437
      DP1413=P21*DP1313+DP21*P1313                                              438
      P1414=P22*P1313                                                           439
      DP1414=13.0*P1413                                                         440
      AOR=AOR*AR                                                                441
      C2=G(14,2)*CP2+H(14,2)*SP2                                                442
      C3=G(14,3)*CP3+H(14,3)*SP3                                                443
      C4=G(14,4)*CP4+H(14,4)*SP4                                                444
      C5=G(14,5)*CP5+H(14,5)*SP5                                                445
      C6=G(14,6)*CP6+H(14,6)*SP6                                                446
      C7=G(14,7)*CP7+H(14,7)*SP7                                                447
      C8=G(14,8)*CP8+H(14,8)*SP8                                                448
      C9=G(14,9)*CP9+H(14,9)*SP9                                                449
      C10=G(14,10)*CP10+H(14,10)*SP10                                           450
      C11=G(14,11)*CP11+H(14,11)*SP11                                           451
      C12=G(14,12)*CP12+H(14,12)*SP12                                           452
      C13=G(14,13)*CP13+H(14,13)*SP13                                           453
      C14=G(14,14)*CP14+H(14,14)*SP14                                           454
      BR=BR-14.0*AOR*(G(14,1)*P141+C2*P142+C3*P143+C4*P144+C5*P145+C6*P1        455
     1146+C7*P147+C8*P148+C9*P149+C10*P1410+C11*P1411+C12*P1412+C13*P1413       456
     2+C14*P1414)                                                               457
      BT=BT+AOR*(G(14,1)*DP141+C2*DP142+C3*DP143+C4*DP144+C5*DP145+C6*DP        458
     1146+C7*DP147+C8*DP148+C9*DP149+C10*DP1410+C11*DP1411+C12*DP1412+C1       459
     123*DP1413+C14*DP1414)                                                     460
      BP=BP-AOR*((G(14,2)*SP2-H(14,2)*CP2)*P142+2.0*(G(14,3)*SP3-H(14,3)        461
     1*CP3)*P143+3.0*(G(14,4)*SP4-H(14,4)*CP4)*P144+4.0*(G(14,5)*SP5-H(1       462
     124,5)*CP5)*P145+5.0*(G(14,6)*SP6-H(14,6)*CP6)*P146+6.0*(G(14,7)*SP7      463
     13-H(14,7)*CP7)*P147+7.0*(G(14,8)*SP8-H(14,8)*CP8)*P148+8.0*(G(14,9)      464
     14*SP9-H(14,9)*CP9)*P149+9.0*(G(14,10)*SP10-H(14,10)*CP10)*P1410+10.      465
     150*(G(14,11)*SP11-H(14,11)*CP11)*P1411+11.0*(G(14,12)*SP12-H(14,12)      466
     16*CP12)*P1412+12.0*(G(14,13)*SP13-H(14,13)*CP13)*P1413+13.0*(G(14,1      467
     174)*SP14-H(14,14)*CP14)*P1414)                                            468
      IF(NMAX-14) 18,18,14                                                      469
C                                                          N=15                 470
14    SP15=(SP8+SP8)*CP8                                                        471
      CP15=(CP8+SP8)*(CP8-SP8)                                                  472
      P151=P21*P141-0.25037037*P131                                             473
      DP151=P21*DP141+DP21*P141-0.25037037*DP131                                474
      P152=P21*P142-0.24888888*P132                                             475
      DP152=P21*DP142+DP21*P142-0.24888888*DP132                                476
      P153=P21*P143-0.24444444*P133                                             477
      DP153=P21*DP143+DP21*P143-0.24444444*DP133                                478
      P154=P21*P144-0.23703703*P134                                             479
      DP154=P21*DP144+DP21*P144-0.23703703*DP134                                480
      P155=P21*P145-0.22666666*P135                                             481
      DP155=P21*DP145+DP21*P145-0.22666666*DP135                                482
      P156=P21*P146-0.21333333*P136                                             483
      DP156=P21*DP146+DP21*P146-0.21333333*DP136                                484
      P157=P21*P147-0.19703703*P137                                             485
      DP157=P21*DP147+DP21*P147-0.19703703*DP137                                486
      P158=P21*P148-0.17777777*P138                                             487
      DP158=P21*DP148+DP21*P148-0.17777777*DP138                                488
```

Table IV (Cont)

```
      P159=P21*P149-0.15555555*P139                                          489
      DP159=P21*DP149+DP21*P149-0.15555555*DP139                             490
      P1510=P21*P1410-0.13037037*P1310                                       491
      DP1510=P21*DP1410+DP21*P1410-0.13037037*DP1310                         492
      P1511=P21*P1411-0.10222222*P1311                                       493
      DP1511=P21*DP1411+DP21*P1411-0.10222222*DP1311                         494
      P1512=P21*P1412-0.07111111*P1312                                       495
      DP1512=P21*DP1412+DP21*P1412-0.07111111*DP1312                         496
      P1513=P21*P1413-0.03703703*P1313                                       497
      DP1513=P21*DP1413+DP21*P1413-0.03703703*DP1313                         498
      P1514=P21*P1414                                                        499
      DP1514=P21*DP1414+DP21*P1414                                           500
      P1515=P22*P1414                                                        501
      DP1515=14.0*P1514                                                      502
      AOR=AOR*AR                                                             503
      C2=G(15,2)*CP2+H(15,2)*SP2                                             504
      C3=G(15,3)*CP3+H(15,3)*SP3                                             505
      C4=G(15,4)*CP4+H(15,4)*SP4                                             506
      C5=G(15,5)*CP5+H(15,5)*SP5                                             507
      C6=G(15,6)*CP6+H(15,6)*SP6                                             508
      C7=G(15,7)*CP7+H(15,7)*SP7                                             509
      C8=G(15,8)*CP8+H(15,8)*SP8                                             510
      C9=G(15,9)*CP9+H(15,9)*SP9                                             511
      C10=G(15,10)*CP10+H(15,10)*SP10                                        512
      C11=G(15,11)*CP11+H(15,11)*SP11                                        513
      C12=G(15,12)*CP12+H(15,12)*SP12                                        514
      C13=G(15,13)*CP13+H(15,13)*SP13                                        515
      C14=G(15,14)*CP14+H(15,14)*SP14                                        516
      C15=G(15,15)*CP15+H(15,15)*SP15                                        517
      BR=BR-15.0*AOR*(G(15,1)*P151+C2*P152+C3*P153+C4*P154+C5*P155+C6*P1     518
     156+C7*P157+C8*P158+C9*P159+C10*P1510+C11*P1511+C12*P1512+C13*P1513     519
     2+C14*P1514+C15*P1515)                                                  520
      BT=BT+AOR*(G(15,1)*DP151+C2*DP152+C3*DP153+C4*DP154+C5*DP155+C6*DP     521
     1156+C7*DP157+C8*DP158+C9*DP159+C10*DP1510+C11*DP1511+C12*DP1512+C1     522
     23*DP1513+C14*DP1514+C15*DP1515)                                        523
      BP=BP-AOR*((G(15,2)*SP2-H(15,2)*CP2)*P152+2.0*(G(15,3)*SP3-H(15,3)     524
     1*CP3)*P153+3.0*(G(15,4)*SP4-H(15,4)*CP4)*P154+4.0*(G(15,5)*SP5-H(1     525
     25,5)*CP5)*P155+5.0*(G(15,6)*SP6-H(15,6)*CP6)*P156+6.0*(G(15,7)*SP7     526
     3-H(15,7)*CP7)*P157+7.0*(G(15,8)*SP8-H(15,8)*CP8)*P158+8.0*(G(15,9)     527
     4*SP9-H(15,9)*CP9)*P159+9.0*(G(15,10)*SP10-H(15,10)*CP10)*P1510+10.     528
     50*(G(15,11)*SP11-H(15,11)*CP11)*P1511+11.0*(G(15,12)*SP12-H(15,12)     529
     6*CP12)*P1512+12.0*(G(15,13)*SP13-H(15,13)*CP13)*P1513+13.0*(G(15,1     530
     74)*SP14-H(15,14)*CP14)*P1514+14.0*(G(15,15)*SP15-H(15,15)*CP15)*P1     531
     8515)                                                                   532
      IF(NMAX-15) 18,18,15                                                   533
C                                                          N=16               534
15    SP16=SP2*CP15+CP2*SP15                                                 535
      CP16=CP2*CP15-SP2*SP15                                                 536
      P161=P21*P151-0.25031928*P141                                          537
      DP161=P21*DP151+DP21*P151-0.25031928*DP141                             538
      P162=P21*P152-0.24904214*P142                                          539
      DP162=P21*DP152+DP21*P152-0.24904214*DP142                             540
      P163=P21*P153-0.24521072*P143                                          541
      DP163=P21*DP153+DP21*P153-0.24521072*DP143                             542
      P164=P21*P154-0.2388250?*P144                                          543
      DP164=P21*DP154+DP21*P154-0.23882503*DP144                             544
      P165=P21*P155-0.22988505*P145                                          545
      DP165=P21*DP155+DP21*P155-0.22988505*DP145                             546
      P166=P21*P156-0.21839080*P146                                          547
      DP166=P21*DP156+DP21*P156-0.21839080*DP146                             548
      P167=P21*P157-0.20434227*P147                                          549
```

Table IV (Cont)

```
      DP167=P21*DP157+DP21*P157-0.20434227*DP147                        550
      P168=P21*P158-0.18773946*P148                                     551
      DP168=P21*DP158+DP21*P158-0.18773946*DP148                        552
      P169=P21*P159-0.16858237*P149                                     553
      DP169=P21*DP159+DP21*P159-0.16858237*DP149                        554
      P1610=P21*P1510-0.14687100*P1410                                  555
      DP1610=P21*DP1510+DP21*P1510-0.14687100*DP1410                    556
      P1611=P21*P1511-0.12260536*P1411                                  557
      DP1611=P21*DP1511+DP21*P1511-0.12260536*DP1411                    558
      P1612=P21*P1512-0.09578544*P1412                                  559
      DP1612=P21*DP1512+DP21*P1512-0.09578544*DP1412                    560
      P1613=P21*P1513-0.06641123*P1413                                  561
      DP1613=P21*DP1513+DP21*P1513-0.06641123*DP1413                    562
      P1614=P21*P1514-0.03448275*P1414                                  563
      DP1614=P21*DP1514+DP21*P1514-0.03448275*DP1414                    564
      P1615=P21*P1515                                                   565
      DP1615=P21*DP1515+DP21*P1515                                      566
      P1616=P22*P1515                                                   567
      DP1616=15.0*P1615                                                 568
      AOR=AOR*AR                                                        569
      C2=G(16,2)*CP2+H(16,2)*SP2                                        570
      C3=G(16,3)*CP3+H(16,3)*SP3                                        571
      C4=G(16,4)*CP4+H(16,4)*SP4                                        572
      C5=G(16,5)*CP5+H(16,5)*SP5                                        573
      C6=G(16,6)*CP6+H(16,6)*SP6                                        574
      C7=G(16,7)*CP7+H(16,7)*SP7                                        575
      C8=G(16,8)*CP8+H(16,8)*SP8                                        576
      C9=G(16,9)*CP9+H(16,9)*SP9                                        577
      C10=G(16,10)*CP10+H(16,10)*SP10                                   578
      C11=G(16,11)*CP11+H(16,11)*SP11                                   579
      C12=G(16,12)*CP12+H(16,12)*SP12                                   580
      C13=G(16,13)*CP13+H(16,13)*SP13                                   581
      C14=G(16,14)*CP14+H(16,14)*SP14                                   582
      C15=G(16,15)*CP15+H(16,15)*SP15                                   583
      C16=G(16,16)*CP16+H(16,16)*SP16                                   584
      BR=BR-16.0*AOR*(G(16,1)*P161+C2*P162+C3*P163+C4*P164+C5*P165+C6*P1 585
     1166+C7*P167+C8*P168+C9*P169+C10*P1610+C11*P1611+C12*P1612+C13*P1613 586
     2+C14*P1614+C15*P1615+C16*P1616)                                   587
      BT=BT+AOR*(G(16,1)*DP161+C2*DP162+C3*DP163+C4*DP164+C5*DP165+C6*DP 588
     1166+C7*DP167+C8*DP168+C9*DP169+C10*DP1610+C11*DP1611+C12*DP1612+C1 589
     123*DP1613+C14*DP1614+C15*DP1615+C16*DP1616)                       590
      BP=BP-AOR*((G(16,2)*SP2-H(16,2)*CP2)*P162+2.0*(G(16,3)*SP3-H(16,3) 591
     1*CP3)*P163+3.0*(G(16,4)*SP4-H(16,4)*CP4)*P164+4.0*(G(16,5)*SP5-H(1 592
     126,5)*CP5)*P165+5.0*(G(16,6)*SP6-H(16,6)*CP6)*P166+6.0*(G(16,7)*SP7 593
     13-H(16,7)*CP7)*P167+7.0*(G(16,8)*SP8-H(16,8)*CP8)*P168+8.0*(G(16,9) 594
     14*SP9-H(16,9)*CP9)*P169+9.0*(G(16,10)*SP10-H(16,10)*CP10)*P1610+10. 595
     150*(G(16,11)*SP11-H(16,11)*CP11)*P1611+11.0*(G(16,12)*SP12-H(16,12) 596
     16*CP12)*P1612+12.0*(G(16,13)*SP13-H(16,13)*CP13)*P1613+13.0*(G(16,1 597
     174)*SP14-H(16,14)*CP14)*P1614+14.0*(G(16,15)*SP15-H(16,15)*CP15)*P1 598
     18615+15.0*(G(16,16)*SP16-H(16,16)*CP16)*P1616)                    599
      IF(NMAX-16) 18,18,16                                              600
C                                                        N=17           601
16    SP17=(SP9+SP9)*CP9                                                602
      CP17=(CP9+SP9)*(CP9-SP9)                                          603
      P171=P21*P161-0.25027808*P151                                     604
      DP171=P21*DP161+DP21*P161-0.25027808*DP151                        605
      P172=P21*P162-0.24916573*P152                                     606
      DP172=P21*DP162+DP21*P162-0.24916573*DP152                        607
      P173=P21*P163-0.24582869*P153                                     608
      DP173=P21*DP163+DP21*P163-0.24582869*DP153                        609
      P174=P21*P164-0.24026696*P154                                     610
```

Table VI (Cont)

1	C	EPOCH= 1946.0			VESTINE		
N	M	G	H	GT	HT	GTT	HTT
2	1	-30570.0000	-0.	9.0000	-0.	-0.	-0.
2	2	-2110.0000	5810.0000	2.0000	1.0000	-0.	-0.
3	1	-1270.0000	-0.	-18.0000	-0.	-0.	-0.
3	2	2960.0000	-1660.0000	0.	-20.0000	-0.	-0.
3	3	1640.0000	540.0000	2.0000	-14.0000	-0.	-0.
4	1	1150.0000	-0.	-0.	-0.	-0.	-0.
4	2	-1730.0000	-520.0000	-0.	-0.	-0.	-0.
4	3	1210.0000	190.0000	-0.	-0.	-0.	-0.
4	4	880.0000	30.0000	-0.	-0.	-0.	-0.
5	1	920.0000	-0.	-0.	-0.	-0.	-0.
5	2	780.0000	140.0000	-0.	-0.	-0.	-0.
5	3	580.0000	-280.0000	-0.	-0.	-0.	-0.
5	4	-380.0000	-80.0000	-0.	-0.	-0.	-0.
5	5	310.0000	-130.0000	-0.	-0.	-0.	-0.
6	1	-240.0000	-0.	-0.	-0.	-0.	-0.
6	2	280.0000	30.0000	-0.	-0.	-0.	-0.
6	3	200.0000	70.0000	-0.	-0.	-0.	-0.
6	4	-60.0000	-20.0000	-0.	-0.	-0.	-0.
6	5	-150.0000	-140.0000	-0.	-0.	-0.	-0.
6	6	-70.0000	80.0000	-0.	-0.	-0.	-0.
7	1	60.0000	-0.	-0.	-0.	-0.	-0.
7	2	110.0000	-70.0000	-0.	-0.	-0.	-0.
7	3	-30.0000	150.0000	-0.	-0.	-0.	-0.
7	4	-260.0000	-20.0000	-0.	-0.	-0.	-0.
7	5	-30.0000	0.	-0.	-0.	-0.	-0.
7	6	30.0000	-10.0000	-0.	-0.	-0.	-0.
7	7	-110.0000	-40.0000	-0.	-0.	-0.	-0.

Vestine (1960) Table 1 correcting G(7,6) from -130. to +30. Secular change from Table 3 for 1942.5.

Table VI (Cont)

1 C EPOCH= 1958.5 NAGATA OGUTI (INTERNAL)

N	M	G	H	GT	HT	GTT	HTT
2	1	-30450.0000	-0.	13.6000	-0.	-0.	-0.
2	2	-2220.0000	5840.0000	5.9000	2.3000	-0.	-0.
3	1	-1510.0000	-0.	-21.5000	-0.	-0.	-0.
3	2	2950.0000	-1940.0000	-1.8000	-16.0000	-0.	-0.
3	3	1490.0000	210.0000	-0.1000	-17.1000	-0.	-0.
4	1	1160.0000	-0.	1.9000	-0.	-0.	-0.
4	2	-1860.0000	-450.0000	-8.2000	8.2000	-0.	-0.
4	3	1230.0000	240.0000	1.1000	3.5000	-0.	-0.
4	4	930.0000	-90.0000	-0.5000	-6.7000	-0.	-0.
5	1	940.0000	-0.	-4.4000	-0.	-0.	-0.
5	2	750.0000	190.0000	2.7000	0.	-0.	-0.
5	3	640.0000	-230.0000	-2.0000	-1.1000	-0.	-0.
5	4	-370.0000	-30.0000	-1.0000	3.5000	-0.	-0.
5	5	290.0000	-200.0000	-2.2000	-2.1000	-0.	-0.
6	1	-190.0000	-0.	2.1000	-0.	-0.	-0.
6	2	340.0000	10.0000	-1.8000	1.8000	-0.	-0.
6	3	210.0000	90.0000	0.2000	1.7000	-0.	-0.
6	4	-80.0000	-90.0000	1.0000	-3.8000	-0.	-0.
6	5	-150.0000	-120.0000	-0.5000	0.7000	-0.	-0.
6	6	-90.0000	100.0000	-0.2000	0.2000	-0.	-0.
7	1	70.0000	-0.	1.7000	-0.	-0.	-0.
7	2	40.0000	-50.0000	2.1000	-0.7000	-0.	-0.
7	3	-10.0000	60.0000	0.1000	-0.2000	-0.	-0.
7	4	-200.0000	20.0000	-0.2000	1.5000	-0.	-0.
7	5	-50.0000	-10.0000	1.5000	-1.1000	-0.	-0.
7	6	-20.0000	-10.0000	0.5000	0.7000	-0.	-0.
7	7	-100.0000	-10.0000	-0.6000	1.0000	-0.	-0.

Nagata and Oguti (1962)

Table VI (Cont)

1	C	EPOCH= 1965.0		LEATON EVANS (X+Y)			
N	M	G	H	GT	HT	GTT	HTT
2	1	-30375.0000	-0.	15.5000	-0.	-0.	-0.
2	2	-2087.0000	5769.0000	8.3000	0.6000	-0.	-0.
3	1	-1648.0000	-0.	-26.6000	-0.	-0.	-0.
3	2	2954.0000	-1995.0000	-1.3000	-11.4000	-0.	-0.
3	3	1579.0000	116.0000	1.3000	-18.2000	-0.	-0.
4	1	1164.0000	-0.	-0.	-0.	-0.	-0.
4	2	-2033.0000	-389.0000	-9.5000	3.2000	-0.	-0.
4	3	1299.0000	230.0000	-1.9000	1.6000	-0.	-0.
4	4	880.0000	-141.0000	-0.6000	-8.5000	-0.	-0.
5	1	930.0000	-0.	0.6000	-0.	-0.	-0.
5	2	811.0000	142.0000	1.0000	3.0000	-0.	-0.
5	3	490.0000	-276.0000	-2.2000	-0.7000	-0.	-0.
5	4	-402.0000	5.0000	0.2000	2.7000	-0.	-0.
5	5	262.0000	-264.0000	-3.0000	-2.7000	-0.	-0.
6	1	-179.0000	-0.	0.8000	-0.	-0.	-0.
6	2	357.0000	30.0000	0.4000	1.9000	-0.	-0.
6	3	248.0000	135.0000	1.6000	2.3000	-0.	-0.
6	4	-20.0000	-123.0000	-0.3000	-1.8000	-0.	-0.
6	5	-171.0000	-100.0000	-1.1000	1.4000	-0.	-0.
6	6	-64.0000	84.0000	1.7000	0.5000	-0.	-0.
7	1	42.0000	-0.	-0.	-0.	-0.	-0.
7	2	55.0000	-18.0000	-0.5000	-2.2000	-0.	-0.
7	3	12.0000	101.0000	1.8000	0.2000	-0.	-0.
7	4	-239.0000	60.0000	1.4000	0.9000	-0.	-0.
7	5	16.0000	-32.0000	0.6000	-2.1000	-0.	-0.
7	6	8.0000	-27.0000	0.2000	-0.1000	-0.	-0.
7	7	-110.0000	-12.0000	-2.2000	-0.2000	-0.	-0.
8	1	77.0000	-0.	-0.	-0.	-0.	-0.
8	2	-56.0000	-47.0000	-0.	-0.	-0.	-0.
8	3	8.0000	-35.0000	-0.	-0.	-0.	-0.
8	4	5.0000	-9.0000	-0.	-0.	-0.	-0.
8	5	-35.0000	2.0000	-0.	-0.	-0.	-0.
8	6	-16.0000	27.0000	-0.	-0.	-0.	-0.
8	7	9.0000	-17.0000	-0.	-0.	-0.	-0.
8	8	2.0000	-24.0000	-0.	-0.	-0.	-0.
9	1	11.0000	-0.	-0.	-0.	-0.	-0.
9	2	23.0000	5.0000	-0.	-0.	-0.	-0.
9	3	-6.0000	-7.0000	-0.	-0.	-0.	-0.
9	4	-17.0000	3.0000	-0.	-0.	-0.	-0.
9	5	5.0000	-20.0000	-0.	-0.	-0.	-0.
9	6	20.0000	8.0000	-0.	-0.	-0.	-0.
9	7	-1.0000	26.0000	-0.	-0.	-0.	-0.
9	8	12.0000	10.0000	-0.	-0.	-0.	-0.
9	9	7.0000	-12.0000	-0.	-0.	-0.	-0.

Private Communication (1964)

Table VI (Cont)

C 1 EPOCH= 1942.5 JONES MELOTTE (OBLATE)

N	M	C	H	GT	HT	GTT	HTT
2	1	30220.0000	-0.	-0.	-0.	-0.	-0.
2	2	2180.0000	-5530.0000	-0.	-0.	-0.	-0.
3	1	1760.0000	-0.	-0.	-0.	-0.	-0.
3	2	-5070.0000	2590.0000	-0.	-0.	-0.	-0.
3	3	-1350.0000	-440.0000	-0.	-0.	-0.	-0.
4	1	-2920.0000	-0.	-0.	-0.	-0.	-0.
4	2	5080.0000	1940.0000	-0.	-0.	-0.	-0.
4	3	-2350.0000	-330.0000	-0.	-0.	-0.	-0.
4	4	-740.0000	-10.0000	-0.	-0.	-0.	-0.
5	1	-3850.0000	-0.	-0.	-0.	-0.	-0.
5	2	-3840.0000	-1430.0000	-0.	-0.	-0.	-0.
5	3	-2340.0000	760.0000	-0.	-0.	-0.	-0.
5	4	870.0000	190.0000	-0.	-0.	-0.	-0.
5	5	-180.0000	100.0000	-0.	-0.	-0.	-0.
6	1	1620.0000	-0.	-0.	-0.	-0.	-0.
6	2	-3420.0000	530.0000	-0.	-0.	-0.	-0.
6	3	-1230.0000	-170.0000	-0.	-0.	-0.	-0.
6	4	330.0000	90.0000	-0.	-0.	-0.	-0.
6	5	330.0000	320.0000	-0.	-0.	-0.	-0.
6	6	50.0000	-40.0000	-0.	-0.	-0.	-0.
7	1	-1420.0000	-0.	-0.	-0.	-0.	-0.
7	2	-540.0000	-190.0000	-0.	-0.	-0.	-0.
7	3	20.0000	2060.0000	-0.	-0.	-0.	-0.
7	4	2050.0000	170.0000	-0.	-0.	-0.	-0.
7	5	180.0000	90.0000	-0.	-0.	-0.	-0.
7	6	-40.0000	50.0000	-0.	-0.	-0.	-0.
7	7	60.0000	20.0000	-0.	-0.	-0.	-0.

Jones and Melotte (1953) Table V (Spheroidal earth)

Table VI (Cont)

1 1 EPOCH= 1960.0 JENSEN CAIN

N	M	G	H	GT	HT	GTT	HTT
2	1	30411.2000	-0.	-0.	-0.	-0.	-0.
2	2	2147.4000	-5798.9000	-0.	-0.	-0.	-0.
3	1	2403.5000	-0.	-0.	-0.	-0.	-0.
3	2	-5125.3000	3312.4000	-0.	-0.	-0.	-0.
3	3	-1338.1000	-157.9000	-0.	-0.	-0.	-0.
4	1	-3151.8000	-0.	-0.	-0.	-0.	-0.
4	2	6213.0000	1487.0000	-0.	-0.	-0.	-0.
4	3	-2489.8000	-407.5000	-0.	-0.	-0.	-0.
4	4	-649.6000	21.0000	-0.	-0.	-0.	-0.
5	1	-4179.4000	-0.	-0.	-0.	-0.	-0.
5	2	-4529.8000	-1182.5000	-0.	-0.	-0.	-0.
5	3	-2179.5000	1000.6000	-0.	-0.	-0.	-0.
5	4	700.8000	43.0000	-0.	-0.	-0.	-0.
5	5	-204.4000	138.5000	-0.	-0.	-0.	-0.
6	1	1625.6000	-0.	-0.	-0.	-0.	-0.
6	2	-3440.7000	-79.6000	-0.	-0.	-0.	-0.
6	3	-1944.7000	-200.0000	-0.	-0.	-0.	-0.
6	4	-60.8000	459.7000	-0.	-0.	-0.	-0.
6	5	277.5000	242.1000	-0.	-0.	-0.	-0.
6	6	69.7000	-121.8000	-0.	-0.	-0.	-0.
7	1	-1952.3000	-0.	-0.	-0.	-0.	-0.
7	2	-485.3000	-575.8000	-0.	-0.	-0.	-0.
7	3	321.2000	-873.5000	-0.	-0.	-0.	-0.
7	4	2141.3000	-340.6000	-0.	-0.	-0.	-0.
7	5	105.1000	-11.8000	-0.	-0.	-0.	-0.
7	6	22.7000	-111.6000	-0.	-0.	-0.	-0.
7	7	111.5000	-32.5000	-0.	-0.	-0.	-0.

Jensen and Cain (1962)

Table IV (Cont)

```
      DP174=P21*DP164+DP21*P164-0.24026696*DP154                                611
      P175=P21*P165-0.23248053*P155                                             612
      DP175=P21*DP165+DP21*P165-0.23248053*DP155                                613
      P176=P21*P166-0.22246941*P156                                             614
      DP176=P21*DP166+DP21*P166-0.22246941*DP156                                615
      P177=P21*P167-0.21023359*P157                                             616
      DP177=P21*DP167+DP21*P167-0.21023359*DP157                                617
      P178=P21*P168-0.19577308*P158                                             618
      DP178=P21*DP168+DP21*P168-0.19577308*DP158                                619
      P179=P21*P169-0.17908787*P159                                             620
      DP179=P21*DP169+DP21*P169-0.17908787*DP159                                621
      P1710=P21*P1610-0.16017797*P1510                                          622
      DP1710=P21*DP1610+DP21*P1610-0.16017797*DP1510                            623
      P1711=P21*P1611-0.13904338*P1511                                          624
      DP1711=P21*DP1611+DP21*P1611-0.13904338*DP1511                            625
      P1712=P21*P1612-0.11568409*P1512                                          626
      DP1712=P21*DP1612+DP21*P1612-0.11568409*DP1512                            627
      P1713=P21*P1613-0.09010011*P1513                                          628
      DP1713=P21*DP1613+DP21*P1613-0.09010011*DP1513                            629
      P1714=P21*P1614-0.06229143*P1514                                          630
      DP1714=P21*DP1614+DP21*P1614-0.06229143*DP1514                            631
      P1715=P21*P1615-0.03225806*P1515                                          632
      DP1715=P21*DP1615+DP21*P1615-0.03225806*DP1515                            633
      P1716=P21*P1616                                                           634
      DP1716=P21*DP1616+DP21*P1616                                              635
      P1717=P22*P1616                                                           636
      DP1717=16.0*P1716                                                         637
      AOR=AOR*AR                                                                638
      C2=G(17,2)*CP2+H(17,2)*SP2                                                639
      C3=G(17,3)*CP3+H(17,3)*SP3                                                640
      C4=G(17,4)*CP4+H(17,4)*SP4                                                641
      C5=G(17,5)*CP5+H(17,5)*SP5                                                642
      C6=G(17,6)*CP6+H(17,6)*SP6                                                643
      C7=G(17,7)*CP7+H(17,7)*SP7                                                644
      C8=G(17,8)*CP8+H(17,8)*SP8                                                645
      C9=G(17,9)*CP9+H(17,9)*SP9                                                646
      C10=G(17,10)*CP10+H(17,10)*SP10                                           647
      C11=G(17,11)*CP11+H(17,11)*SP11                                           648
      C12=G(17,12)*CP12+H(17,12)*SP12                                           649
      C13=G(17,13)*CP13+H(17,13)*SP13                                           650
      C14=G(17,14)*CP14+H(17,14)*SP14                                           651
      C15=G(17,15)*CP15+H(17,15)*SP15                                           652
      C16=G(17,16)*CP16+H(17,16)*SP16                                           653
      C17=G(17,17)*CP17+H(17,17)*SP17                                           654
      BR=BR-17.0*AOR*(G(17,1)*P171+C2*P172+C3*P173+C4*P174+C5*P175+C6*P1        655
     1176+C7*P177+C8*P178+C9*P179+C10*P1710+C11*P1711+C12*P1712+C13*P1713       656
     2+C14*P1714+C15*P1715+C16*P1716+C17*P1717)                                 657
      BT=BT+AOR*(G(17,1)*DP171+C2*DP172+C3*DP173+C4*DP174+C5*DP175+C6*DP        658
     1176+C7*DP177+C8*DP178+C9*DP179+C10*DP1710+C11*DP1711+C12*DP1712+C1       659
     23*DP1713+C14*DP1714+C15*DP1715+C16*DP1716+C17*DP1717)                     660
      BP=BP-AOR*((G(17,2)*SP2-H(17,2)*CP2)*P172+2.0*(G(17,3)*SP3-H(17,3)        661
     1*CP3)*P173+3.0*(G(17,4)*SP4-H(17,4)*CP4)*P174+4.0*(G(17,5)*SP5-H(1       662
     27,5)*CP5)*P175+5.0*(G(17,6)*SP6-H(17,6)*CP6)*P176+6.0*(G(17,7)*SP7       663
     3-H(17,7)*CP7)*P177+7.0*(G(17,8)*SP8-H(17,8)*CP8)*P178+8.0*(G(17,9)       664
     4*SP9-H(17,9)*CP9)*P179+9.0*(G(17,10)*SP10-H(17,10)*CP10)*P1710+10.       665
     50*(G(17,11)*SP11-H(17,11)*CP11)*P1711+11.0*(G(17,12)*SP12-H(17,12)       666
     6*CP12)*P1712+12.0*(G(17,13)*SP13-H(17,13)*CP13)*P1713+13.0*(G(17,1       667
     74)*SP14-H(17,14)*CP14)*P1714+14.0*(G(17,15)*SP15-H(17,15)*CP15)*P1       668
     8715+15.0*(G(17,16)*SP16-H(17,16)*CP16)*P1716+16.0*(G(17,17)*SP17-H       669
     9(17,17)*CP17)*P1717)                                                      670
      IF(NMAX-17) 18,18,17                                                      671
```

Table IV (Cont)

```
C                                                                  N=18          672
17    SP18=SP2*CP17+CP2*SP17                                                     673
      CP18=CP2*CP17-SP2*SP17                                                     674
      P181=P21*P171-0.25024437*P161                                              675
      DP181=P21*DP171+DP21*P171-0.25024437*DP161                                 676
      P182=P21*P172-0.24926686*P162                                              677
      DP182=P21*DP172+DP21*P172-0.24926686*DP162                                 678
      P183=P21*P173-0.24633431*P163                                              679
      DP183=P21*DP173+DP21*P173-0.24633431*DP163                                 680
      P184=P21*P174-0.24144672*P164                                              681
      DP184=P21*DP174+DP21*P174-0.24144672*DP164                                 682
      P185=P21*P175-0.23460410*P165                                              683
      DP185=P21*DP175+DP21*P175-0.23460410*DP165                                 684
      P186=P21*P176-0.22580645*P166                                              685
      DP186=P21*DP176+DP21*P176-0.22580645*DP166                                 686
      P187=P21*P177-0.21505376*P167                                              687
      DP187=P21*DP177+DP21*P177-0.21505376*DP167                                 688
      P188=P21*P178-0.20234604*P168                                              689
      DP188=P21*DP178+DP21*P178-0.20234604*DP168                                 690
      P189=P21*P179-0.18768328*P169                                              691
      DP189=P21*DP179+DP21*P179-0.18768328*DP169                                 692
      P1810=P21*P1710-0.17106549*P1610                                           693
      DP1810=P21*DP1710+DP21*P1710-0.17106549*DP1610                             694
      P1811=P21*P1711-0.15249266*P1611                                           695
      DP1811=P21*DP1711+DP21*P1711-0.15249266*DP1611                             696
      P1812=P21*P1712-0.13196480*P1612                                           697
      DP1812=P21*DP1712+DP21*P1712-0.13196480*DP1612                             698
      P1813=P21*P1713-0.10948191*P1613                                           699
      DP1813=P21*DP1713+DP21*P1713-0.10948191*DP1613                             700
      P1814=P21*P1714-0.08504398*P1614                                           701
      DP1814=P21*DP1714+DP21*P1714-0.08504398*DP1614                             702
      P1815=P21*P1715-0.05865102*P1615                                           703
      DP1815=P21*DP1715+DP21*P1715-0.05865102*DP1615                             704
      P1816=P21*P1716-0.03030303*P1616                                           705
      DP1816=P21*DP1716+DP21*P1716-0.03030303*DP1616                             706
      P1817=P21*P1717                                                            707
      DP1817=P21*DP1717+DP21*P1717                                               708
      P1818=P22*P1717                                                            709
      DP1818=17.0*P1817                                                          710
      AOR=AOR*AR                                                                 711
      C2=G(18,2)*CP2+H(18,2)*SP2                                                 712
      C3=G(18,3)*CP3+H(18,3)*SP3                                                 713
      C4=G(18,4)*CP4+H(18,4)*SP4                                                 714
      C5=G(18,5)*CP5+H(18,5)*SP5                                                 715
      C6=G(18,6)*CP6+H(18,6)*SP6                                                 716
      C7=G(18,7)*CP7+H(18,7)*SP7                                                 717
      C8=G(18,8)*CP8+H(18,8)*SP8                                                 718
      C9=G(18,9)*CP9+H(18,9)*SP9                                                 719
      C10=G(18,10)*CP10+H(18,10)*SP10                                            720
      C11=G(18,11)*CP11+H(18,11)*SP11                                            721
      C12=G(18,12)*CP12+H(18,12)*SP12                                            722
      C13=G(18,13)*CP13+H(18,13)*SP13                                            723
      C14=G(18,14)*CP14+H(18,14)*SP14                                            724
      C15=G(18,15)*CP15+H(18,15)*SP15                                            725
      C16=G(18,16)*CP16+H(18,16)*SP16                                            726
      C17=G(18,17)*CP17+H(18,17)*SP17                                            727
      C18=G(18,18)*CP18+H(18,18)*SP18                                            728
      BR=BR-18.0*AOR*(G(18,1)*P181+C2*P182+C3*P183+C4*P184+C5*P185+C6*P1         729
     186+C7*P187+C8*P188+C9*P189+C10*P1810+C11*P1811+C12*P1812+C13*P1813         730
     2+C14*P1814+C15*P1815+C16*P1816+C17*P1817+C18*P1818)                        731
      BT=BT+AOR*(G(18,1)*DP181+C2*DP182+C3*DP183+C4*DP184+C5*DP185+C6*DP         732
```

Table IV (Cont)

```
     1186+C7*DP187+C8*DP188+C9*DP189+C10*DP1810+C11*DP1811+C12*DP1812+C1      733
     23*DP1813+C14*DP1814+C15*DP1815+C16*DP1816+C17*DP1817+C18*DP1818)       734
      BP=BP-AUR*((G(18,2)*SP2-H(18,2)*CP2)*P182+2.0*(G(18,3)*SP3-H(18,3)     735
     1*CP3)*P183+3.0*(G(18,4)*SP4-H(18,4)*CP4)*P184+4.0*(G(18,5)*SP5-H(1     736
     28,5)*CP5)*P185+5.0*(G(18,6)*SP6-H(18,6)*CP6)*P186+6.0*(G(18,7)*SP7     737
     3-H(13,7)*CP7)*P187+7.0*(G(18,8)*SP8-H(18,8)*CP8)*P188+8.0*(G(18,9)     738
     4*SP9-H(18,9)*CP9)*P189+9.0*(G(18,10)*SP10-H(18,10)*CP10)*P1810+10.     739
     50*(G(18,11)*SP11-H(18,11)*CP11)*P1811+11.0*(G(18,12)*SP12-H(18,12)     740
     6*CP12)*P1812+12.0*(G(18,13)*SP13-H(18,13)*CP13)*P1813+13.0*(G(18,1     741
     74)*SP14-H(18,14)*CP14)*P1814+14.0*(G(18,15)*SP15-H(18,15)*CP15)*P1     742
     8815+15.0*(G(18,16)*SP16-H(18,16)*CP16)*P1816+16.0*(G(18,17)*SP17-H     743
     9(18,17)*CP17)*P1817+17.0*(G(18,18)*SP18-H(18,18)*CP18)*P1818)          744
18    BP=BP/P22                                                              745
      B=SQRT(BR*BR+BT*BT+BP*BP)                                              746
      RETURN                                                                 747
      END                                                                    748
```

Table V

```
      SUBROUTINE CONVRT (G,H,NMAX,K)                                    1
C     K=1 CONVERTS SCHMIDT TO GAUSS. K=2 CONVERTS GAUSS TO SCHMIDT      2
      DIMENSION G(18,18),H(18,18),S(18,18,2)                            3
      S(1,1,1)=-1.                                                      4
      DO 1 N=2,NMAX                                                     5
      S(N,1,1)=S(N-1,1,1)*FLOAT(2*N-3)/FLOAT(N-1)                       6
      J=2                                                               7
      DO 1 M=2,N                                                        8
      S(N,M,1)=S(N,M-1,1)*SQRT(FLOAT((N-M+1)*J)/FLOAT(N+M-2))           9
1     J=1                                                               10
      DO 2 N=2,NMAX                                                     11
      DO 2 M=1,N                                                        12
      S(N,M,2)=1./S(N,M,1)                                              13
      G(N,M)=G(N,M)*S(N,M,K)                                            14
2     H(N,M)=H(N,M)*S(N,M,K)                                            15
      RETURN                                                            16
      END                                                               17
```

Table VI

1	C	EPOCH= 1958.0		ADAM, ET. AL.			
N	M	G	H	GT	HT	GTT	HTT
2	1	-30610.0000	-0.	11.0000	-0.	-0.	-0.
2	2	-2090.0000	5730.0000	6.0000	1.0000	-0.	-0.
3	1	-1380.0000	-0.	-22.0000	-0.	-0.	-0.
3	2	2910.0000	-1880.0000	0.	-12.0000	-0.	-0.
3	3	1890.0000	-40.0000	5.0000	-20.0000	-0.	-0.
4	1	950.0000	-0.	3.0000	-0.	-0.	-0.
4	2	-1740.0000	-600.0000	-12.0000	-1.0000	-0.	-0.
4	3	1110.0000	370.0000	0.	5.0000	-0.	-0.
4	4	660.0000	30.0000	-4.0000	-8.0000	-0.	-0.
5	1	1130.0000	-0.	-2.0000	-0.	-0.	-0.
5	2	950.0000	260.0000	4.0000	4.0000	-0.	-0.
5	3	620.0000	-340.0000	-2.0000	-1.0000	-0.	-0.
5	4	-110.0000	-40.0000	1.0000	1.0000	-0.	-0.
5	5	410.0000	-330.0000	-2.0000	-3.0000	-0.	-0.
6	1	-130.0000	-0.	-0.	-0.	-0.	-0.
6	2	100.0000	-380.0000	-0.	-0.	-0.	-0.
6	3	170.0000	110.0000	-0.	-0.	-0.	-0.
6	4	-200.0000	-130.0000	-0.	-0.	-0.	-0.
6	5	-440.0000	-160.0000	-0.	-0.	-0.	-0.
6	6	-330.0000	40.0000	-0.	-0.	-0.	-0.
7	1	-120.0000	-0.	-0.	-0.	-0.	-0.
7	2	120.0000	40.0000	-0.	-0.	-0.	-0.
7	3	20.0000	290.0000	-0.	-0.	-0.	-0.
7	4	-380.0000	30.0000	-0.	-0.	-0.	-0.
7	5	20.0000	200.0000	-0.	-0.	-0.	-0.
7	6	-250.0000	360.0000	-0.	-0.	-0.	-0.
7	7	-160.0000	70.0000	-0.	-0.	-0.	-0.

Adam, et al (1962) Table 4

Adam et al (1963) Secular change from Appendix for 61 observatories 1954-1959.

Table VI (Cont)

C C EPOCH= 1960.0 APRIL 64 COEFFICIENTS

N	M	G	H	GT	HT	GTT	HTT
2	1	-30426.3718	0.	18.9272	0.	-0.	-0.
2	2	-2173.5851	5761.1544	7.3484	-1.8917	-0.	-0.
3	1	-1547.8441	0.	-24.8107	0.	-0.	-0.
3	2	2999.8304	-1949.1511	-0.8373	-14.0435	-0.	-0.
3	3	1574.2527	201.4746	0.7795	-17.6824	-0.	-0.
4	1	1322.6534	0.	-0.4255	0.	-0.	-0.
4	2	-2008.9058	-441.7920	-10.4983	1.9260	-0.	-0.
4	3	1275.0282	232.8557	3.3926	3.9573	-0.	-0.
4	4	877.2960	-117.9688	-1.9430	-8.9673	-0.	-0.
5	1	956.7080	0.	0.8136	0.	-0.	-0.
5	2	796.9928	149.2406	5.3684	-0.9192	-0.	-0.
5	3	527.2557	-266.3760	-1.9078	-1.6638	-0.	-0.
5	4	-400.3606	-3.6335	-0.1560	3.1923	-0.	-0.
5	5	272.6586	-262.0834	0.8262	-5.4794	-0.	-0.
6	1	-241.0096	0.	3.5132	0.	-0.	-0.
6	2	352.9445	0.1744	-0.6879	1.8192	-0.	-0.
6	3	231.1537	124.3802	2.4824	2.9407	-0.	-0.
6	4	-33.1023	-103.5910	0.6225	-0.7699	-0.	-0.
6	5	-146.6943	-98.2729	0.0353	-0.3726	-0.	-0.
6	6	-78.6225	75.4376	1.5714	-0.2396	-0.	-0.
7	1	57.6125	-0.	-0.	-0.	-0.	-0.
7	2	70.6926	5.9803	-0.	-0.	-0.	-0.
7	3	20.4417	85.5440	-0.	-0.	-0.	-0.
7	4	-240.9729	58.2674	-0.	-0.	-0.	-0.
7	5	-19.1271	-18.0492	-0.	-0.	-0.	-0.
7	6	-0.4148	-24.8694	-0.	-0.	-0.	-0.
7	7	-100.4090	4.5572	-0.	-0.	-0.	-0.
8	1	89.9445	-0.	-0.	-0.	-0.	-0.
8	2	-47.2579	-51.1593	-0.	-0.	-0.	-0.
8	3	-2.1939	-22.4600	-0.	-0.	-0.	-0.
8	4	-25.8317	6.4347	-0.	-0.	-0.	-0.
8	5	-11.3775	-37.7228	-0.	-0.	-0.	-0.
8	6	26.0820	43.7049	-0.	-0.	-0.	-0.
8	7	5.6310	-3.6613	-0.	-0.	-0.	-0.
8	8	5.8776	-26.2983	-0.	-0.	-0.	-0.

Daniels and Cain (1964)

Table VI (Cont)

1	C	EPOCH= 1955.0		FINCH LEATON			
N	M	G	H	GT	HT	GTT	HTT
2	1	-3055C.CCCC	-0.	1C.CCCC	-0.	-0.	-C.
2	2	-2270.CCCC	5900.0000	1C.CCCC	-0.	-0.	-C.
3	1	-152C.CCCC	-0.	-20.5CCC	-0.	-0.	-0.
3	2	3030.CCCC	-1900.C000	0.	-16.5000	-0.	-C.
3	3	1580.CCCC	240.0000	-C.	-13.CCCC	-0.	-C.
4	1	1180.CCCC	-0.	-0.6667	-C.	-0.	-C.
4	2	-1910.CCCC	-450.0000	-12.6667	7.3333	-0.	-0.
4	3	1260.CCCC	290.CCCC	1.3333	4.CCCC	-0.	-C.
4	4	910.CCCC	-90.CCCC	-2.3333	-10.CCCC	-0.	-C.
5	1	950.CCCC	-0.	1.75CC	-0.	-0.	-C.
5	2	800.CCCC	150.0000	3.CCCC	-2.5000	-0.	-0.
5	3	580.CCCC	-310.CCCC	-3.25CC	-2.CCCC	-0.	-0.
5	4	-380.CCCC	-40.0000	1.CCCC	1.5000	-0.	-C.
5	5	310.CCCC	-170.0000	1.25CC	-6.CCCC	-0.	-0.
6	1	-270.CCCC	-0.	-1.6000	-C.	-0.	-0.
6	2	320.CCCC	20.CCCC	C.2CCC	5.2CCC	-0.	-C.
6	3	200.CCCC	100.CCCC	2.80CC	1.2CCC	-0.	-C.
6	4	-40.CCCC	-50.0000	0.20CC	-2.6000	-0.	-0.
6	5	-150.CCCC	-140.0000	-1.0000	1.6000	-0.	-0.
6	6	-70.CCCC	90.CCCC	0.40CC	0.4000	-0.	-C.
7	1	100.CCCC	-0.	C.1667	-0.	-0.	-C.
7	2	50.CCCC	-20.0000	1.CCCC	-3.3333	-0.	-C.
7	3	20.CCCC	110.0000	1.1667	-0.8333	-0.	-0.
7	4	-240.CCCC	0.	1.CCCC	2.CCCC	-0.	-C.
7	5	-30.CCCC	-10.0000	-C.5CCC	-1.667C	-0.	-C.
7	6	0.	-30.0000	0.1667	2.5000	-0.	-C.
7	7	-110.0000	-10.0000	-1.6670	-0.	-C.	-C.

Finch and Leaton (1960)

Leaton (1962)

Table VI (Cont)

1	C	EPOCH= 1960.0		FOUGERE			
N	M	G	H	GT	HT	GTT	HTT
2	1	-30509.0000	0.	-0.	-0.	-0.	-0.
2	2	-2181.0000	5841.0000	-0.	-0.	-0.	-0.
3	1	-1464.0000	0.	-0.	-0.	-0.	-0.
3	2	2971.0000	-1988.0000	-0.	-0.	-0.	-0.
3	3	1673.0000	198.0000	-0.	-0.	-0.	-0.
4	1	1147.0000	0.	-0.	-0.	-0.	-0.
4	2	-2012.0000	-390.0000	-0.	-0.	-0.	-0.
4	3	1151.0000	291.0000	-0.	-0.	-0.	-0.
4	4	831.0000	-174.0000	-0.	-0.	-0.	-0.
5	1	997.0000	0.	-0.	-0.	-0.	-0.
5	2	863.0000	173.0000	-0.	-0.	-0.	-0.
5	3	583.0000	-292.0000	-0.	-0.	-0.	-0.
5	4	-312.0000	69.0000	-0.	-0.	-0.	-0.
5	5	237.0000	-206.0000	-0.	-0.	-0.	-0.
6	1	-267.0000	0.	-0.	-0.	-0.	-0.
6	2	272.0000	-39.0000	-0.	-0.	-0.	-0.
6	3	251.0000	161.0000	-0.	-0.	-0.	-0.
6	4	-52.0000	-127.0000	-0.	-0.	-0.	-0.
6	5	-113.0000	-147.0000	-0.	-0.	-0.	-0.
6	6	-8.0000	88.0000	-0.	-0.	-0.	-0.
7	1	48.0000	0.	-0.	-0.	-0.	-0.
7	2	131.0000	1.0000	-0.	-0.	-0.	-0.
7	3	-15.0000	62.0000	-0.	-0.	-0.	-0.
7	4	-224.0000	38.0000	-0.	-0.	-0.	-0.
7	5	13.0000	19.0000	-0.	-0.	-0.	-0.
7	6	-71.0000	4.0000	-0.	-0.	-0.	-0.
7	7	-67.0000	-57.0000	-0.	-0.	-0.	-0.
8	1	104.0000	0.	-0.	-0.	-0.	-0.
8	2	-64.0000	-69.0000	-0.	-0.	-0.	-0.
8	3	8.0000	7.0000	-0.	-0.	-0.	-0.
8	4	-18.0000	37.0000	-0.	-0.	-0.	-0.
8	5	-42.0000	22.0000	-0.	-0.	-0.	-0.
8	6	55.0000	8.0000	-0.	-0.	-0.	-0.
8	7	-6.0000	62.0000	-0.	-0.	-0.	-0.
8	8	-33.0000	3.0000	-0.	-0.	-0.	-0.
9	1	-16.0000	0.	-0.	-0.	-0.	-0.
9	2	6.0000	-6.0000	-0.	-0.	-0.	-0.
9	3	13.0000	-21.0000	-0.	-0.	-0.	-0.
9	4	-5.0000	-1.0000	-0.	-0.	-0.	-0.
9	5	33.0000	-70.0000	-0.	-0.	-0.	-0.
9	6	-10.0000	5.0000	-0.	-0.	-0.	-0.
9	7	-7.0000	-11.0000	-0.	-0.	-0.	-0.
9	8	61.0000	0.	-0.	-0.	-0.	-0.
9	9	0.	0.	-0.	-0.	-0.	-0.

Fougere (1964)